LECTURE NOTES

ON

THEORY OF MACHINE

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UNIT- 1

SIMPLE MECHANISM

Theory of machines is that branch of science which deals with the study of relative motion between various parts of machine and the forces which act on them.

Statics:- It deals with the forces and their effect while acting upon the bodies at rest.

Dynamics:- It deals with the forces and their effect while acting upon the bodies when in motion.

Dynamics may be further subdevided in to two types. (i) kinetics
(ii) kinematics

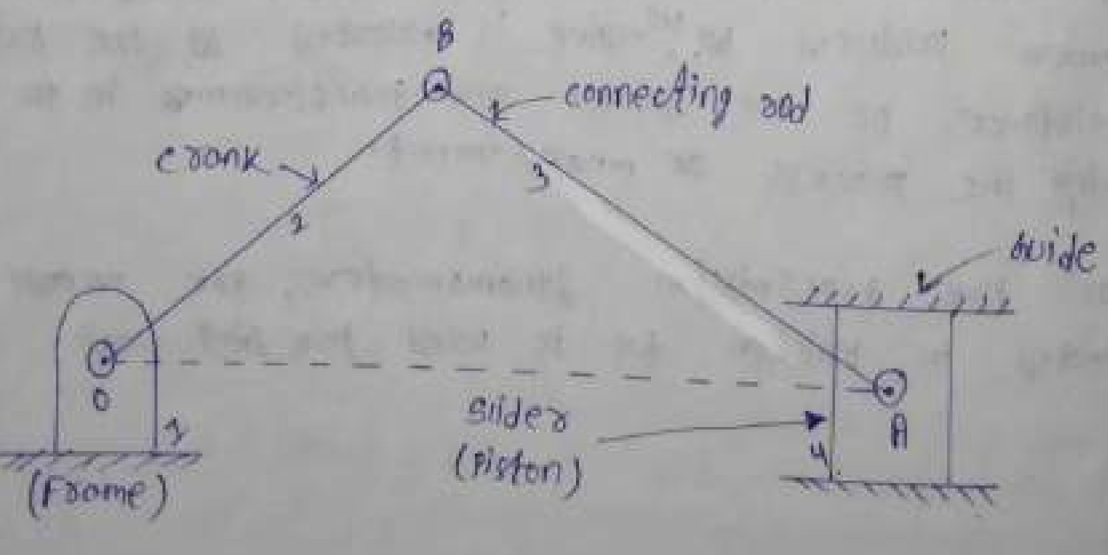
(i) kinetics:- kinetics deals with the forces acting on the moving bodies.

(ii) kinematics:- It deals with the relative motion between various machine parts.

various forces involved in the motion are not considered

Mechanism

A combination of rigid bodies which are so shaped and connected, that they move upon each other with definite relative motion is known as a mechanism.



The figure shown slider crank mechanism. It is the combination of rigid or body namely crank, connecting rod and slider. They are so shaped and connected that they move upon each other with definite relative motion. It converts reciprocating motion of slider in to rotary motion of the crank or vice versa.

Machine :-

A machine is a mechanism or combination of mechanism which do not only imparts definite motion available mechanical energy in to some kind of useful energy.

The slider crank mechanism will become a machine when it is used in automobile engine by adding valve mechanism etc.

Link :-

A link is defined as a member or combination of member, connecting other members and having motion relative to them.

A slider crank mechanism consider of following four links.

(i) Frame
(ii) crank
(iii) connecting rod
(iv) slider

Kinematic pair :-

A Joint of two links relative motion between them is known as kinematic pair. In a slider crank mechanism link 2 rotates relative to link, hence link 1 and link 2 is a kinematic pair. similarly link 2 is having motion relative to link 3 and hence link 2 and link 3 is kinematic pair. Hence link 3, 4 and link 4, 1 consitute kinematic pair.

Classification of kinematic pair

According to nature of contact :-

(a) Lower pair
(b) Higher pair

(a) Lower pair :-

A kinematic pair is known as lower pair. If the two links has surface contact or area contact between them. Also the contact surface of the two links are similar.

Ex:- Shaft rotating in a bearing.

Nut turning on a screw.

(b) Higher pair :-

If the two links has a point or line contact between them, then the kinematic pair is known as higher pair. The contact surface of the two links are not similar.

Ex:- cam and follower.

Wheel rolling on a surface.

Kinematic chain

A kinematic chain is defined as the combination of kinematic pairs, joined in such a way that each link forms a pair of two pairs and the motion of each relative to other is definite.

When the kinematic pairs are coupled in such a way that the last link is joined to the first link to transmit definite motion, is called kinematic chain.

In a slider crank mechanism link is connected to link 2 and also to link 4. Hence link 1,2 and link 1,4 forms a kinematic pair, hence link 1 forms a part of two pairs similarly link 2 forms a part of two pairs (2,1 and 2,3) similarly link 3 and link 4 each frames a part of two pairs. Hence the slider crank mechanism each link form a part of two pairs and motion of each relative to other definite. Hence the total combination of them links is a kinematic chain.

Let, L = No of links
p = No of pairs
J = No of joint

In a kab kinematic chain

$L = 2p - 4$

$J = \frac{3}{2}L - 2$

L.H.S > R.H.S (then chain is locked)

L.H.S = R.H.S (chain is constrained)

L.H.S < R.H.S (chain is unconstrained)

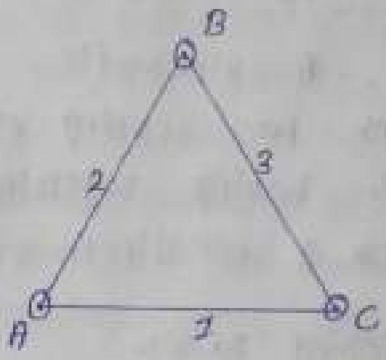
① A three links chain with three joints is shown in fig. prove that the chain is locked.

Ans: Given data,

No of Joint (J) = 3

No of link (L) = 3

No of pair (P) = 3



$L = 2p - 4$

$\Rightarrow 3 = 2 \times 3 - 4$

$\Rightarrow 3 = 6 - 4$

$\Rightarrow 3 = 2$

$\therefore$ L.H.S > R.H.S

$J = \frac{3}{2}L - 2$

$\Rightarrow 3 = \frac{3}{2} \times 3 - 2$

$\Rightarrow 3 = \frac{9}{2} - 2$

$\Rightarrow 3 = \frac{9-4}{2} = \frac{5}{2}$

$\therefore$ L.H.S > R.H.S

$\therefore$ chain is locked.

According to the type of relative motion between two link

(i) sliding pair
(ii) Turning pair
(iii) Rolling pair
(iv) screw pair / Helical pair
(v) spherical pair

(i) sliding pair :-

A kinematic pair is known as sliding pair if the two links have a sliding motion relative to each other. Link 4 and link 1 are having sliding motion relative to each other.

(ii) Turning pair :-

A kinematic pair known as ~~turing~~ turning pair if one link has turning or revolving motions. relative to other, link 2 is having revolving motion relative to link 1, hence link 2 and link 1 constitute turning pair.

(iii) Rolling pair :-

A kinematic pair is known as rolling pair if one link has a rolling wheel on a flat surface form a rolling pair.

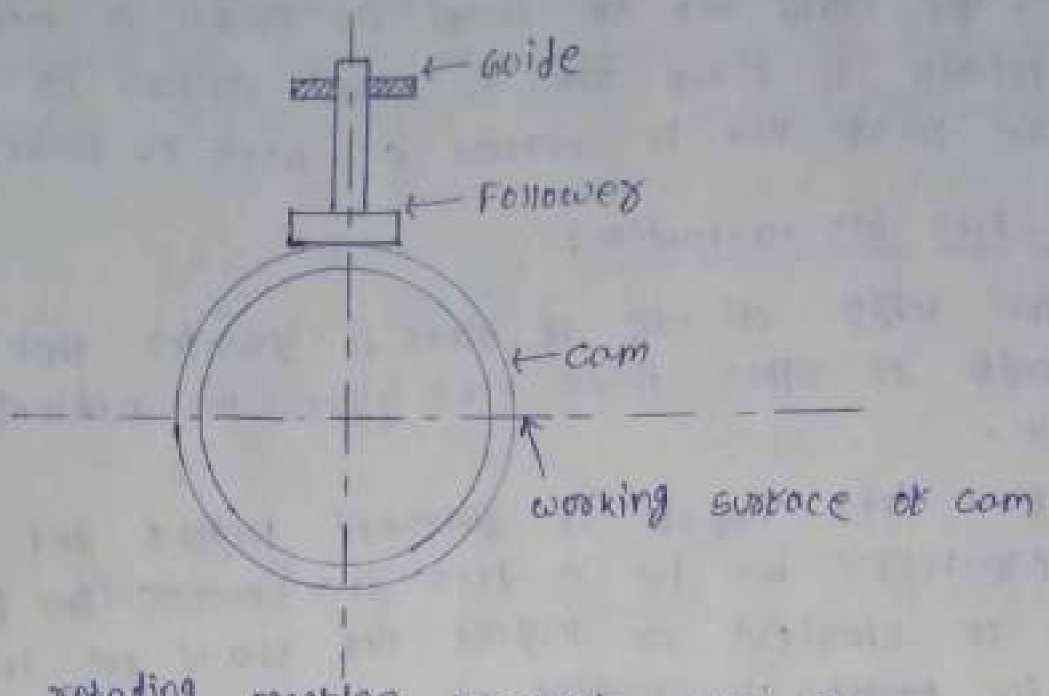
(iv) screw pair :-

A kinematic pair is known as screw pair if the two links have a turning as well as sliding motion between them. The lead screw and then of a lathe is a screw pair.

(v) spherical pair :-

A kinematic pair is known as spherical pair if one link in the form of a sphere turns over a fixed link the ball and socket joint is a spherical pair.

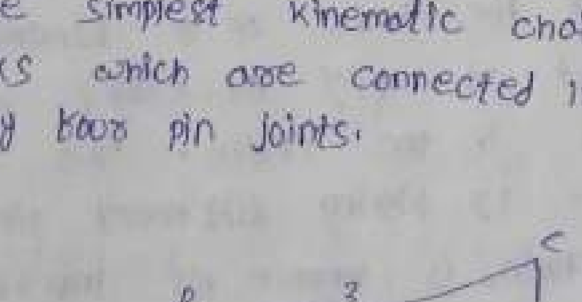
Cam and Follower



A rotating machine element, which gives receprocating or oscillating motion to a second element is known as a cam. The second element is called as follower. The cam rotates usually at constant speed and drives the follower whose motion depends upon the shape of cam. Almost always this cam is a driver and follower is driven. The cam are commonly used in internal combustion engine in machine tools and in automatic machine tools etc.

Four Bar Mechanism

This is the simplest kinematic chain, it consists of four rigid links which are connected in the form of a quadrilateral by four pin joints.



It consists of four turning pairs link 1 and link 2 forms first turning pair, link 2 and link 3 forms second, link 4 and link 1 form fourth turning pair.

A link that makes complete revolution is known as crank. The fixed link is called as frame of mechanism. The link opposite to fixed link is called coupler or connecting rod. The fourth link is unknow a lever or rocker.

For a four bar mechanism :-

(i) If the length of one of links is greater than the sum of length of other three links, four bar mechanism is not possible.

(ii) The four links may be of different lengths but according to Grashoff's law for a four bar mechanism the sum of length of shortest or longest link should not be greater than the ~~length of shortest or~~ sum of length of the remaining two links for continuously relative motion between the two links.

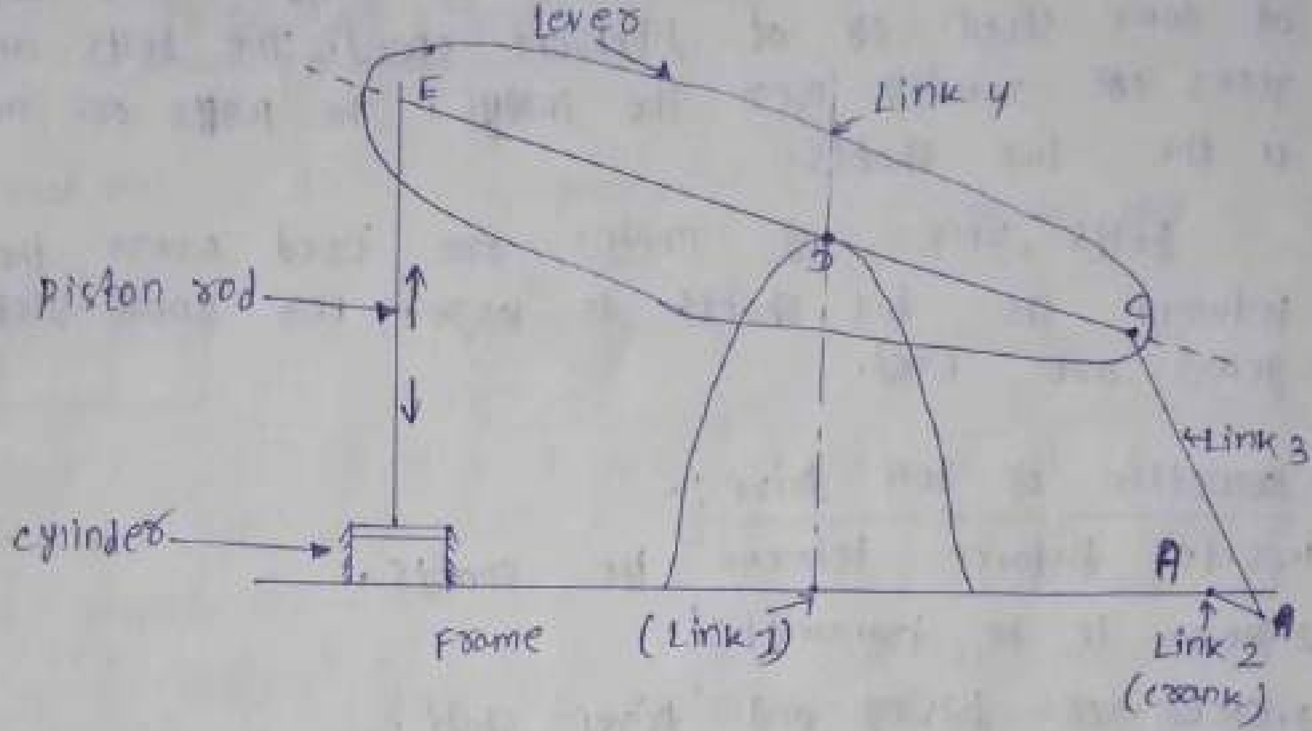
(iii) one of links should make a complete revolution relative to other three links. The mechanism of which no link makes a complete revoluting is not useful.

Inversion of Mechanism

Mechanism is a kinematic chain in which one link as fixed. By fixing the links of a kinematic chain one at a time, we get as many different mechanism as the no of links in the chain. This method of obtaining different mechanism by fixing different links of the same kinematic chain, is known as inversion of mechanism.

Crank-lever Mechanism / Beam engine

It is part of the mechanism of a beam engine which consists of four link. in this mechanism when the crank, rotates about the fixed center 'A'. The lever oscillate about a fixed center 'D'. The ends of the lever CDE is connected to a piston rod which reciprocates due to rotation of crank. The purpose of the mechanism is to convert rotary motion in to reciprocating motion.



UNIT-2

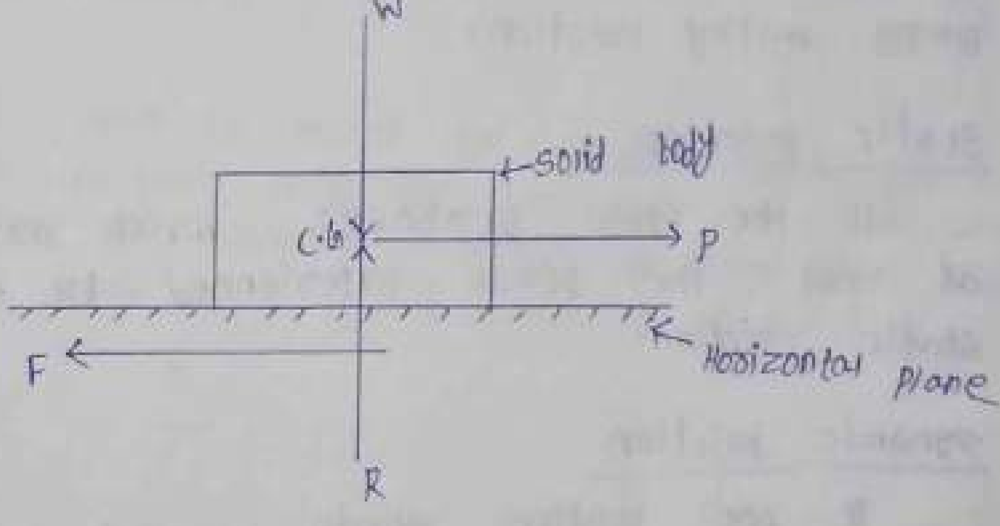
21.02.23

FRICTION

When a solid body slides over a stationary solid body. a force is exerted at the surface of contact by the stationary body on moving body. This force is called as force of friction and is always acting in the direction opposite to the direction of motion.

or

The property of body by virtue is which a force is exerted by a stationary body on moving body to resist the motion of moving body. is called friction.



Let,

W = weight of body acting downward through CG.

R = Normal reaction of body acting through CG upward.

P = Force acting on the body through C.G and parallel to the horizontal surface.

μ = co-efficient of friction.

ϕ = angle of friction.

Co-efficient of friction (μ) :-

It is defined as the ratio of the limiting force of friction to the normal reaction.

$$\mu = \frac{\text{Limiting force of friction}}{\text{Normal reaction}}$$

$$\boxed{\mu = \frac{F}{R}}$$

Limiting force of friction (F) :-

The maximum value of frictional force which comes to play, when a body just begin to slide over the surface of other body, is known as limiting force of friction. or simply limiting friction.

Static friction

If the two surfaces, which are in contact are at rest, the force experienced by one surface is called static friction.

Dynamic friction

If one surface starts moving and other is at rest. the force experienced by the moving surface is called dynamic friction.

Depending on conditions of surfaces the friction is classified as :-

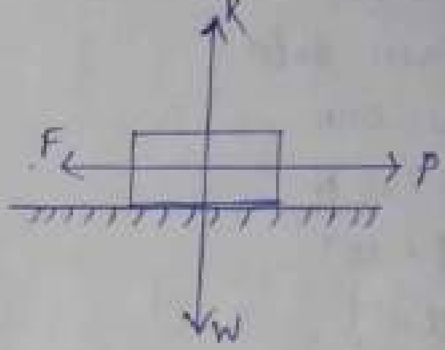
(i) Dry friction (here no lubricant is use)
(ii) Greasy friction or skin friction or boundary friction (here thin lubricant is use)
(iii) Film friction or viscous friction

problem-1

A body of weight 100 N replaced on rough horizontal plane determine the co-efficient of friction if horizontal force of 60 N just causes the body to slide over the horizontal plane.

solve

Given data



$W = 100N \quad (W = R)$

$P = 60N \quad (P = F)$

$\mu = ?$

We know, $\mu = \frac{F}{R}$

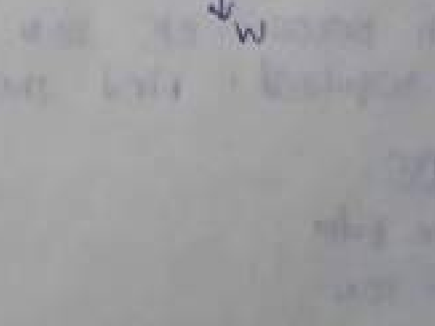
$= \frac{60}{100} = 0.6.$

problem-2

A body of weight 200N is placed on rough horizontal plane if the coefficient friction between the body and horizontal plane 0.3, Determine the horizontal force required just slide the body horizontal plane.

solution

Given data



$W = 200 N \quad (W = R)$

$\mu = 0.3$

We know, $\mu = \frac{F}{R}$

$F = \mu R$

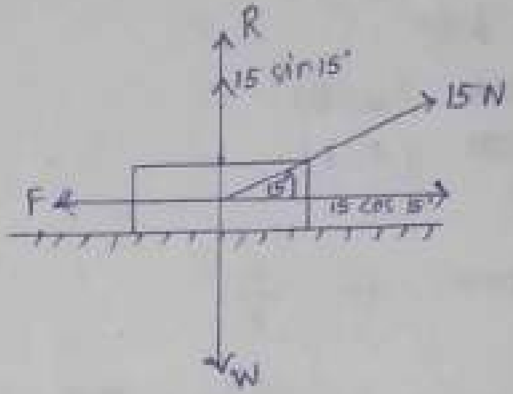
$= \mu W$

$= 0.3 \times 200 = 60N.$

Problem - 3

The force required to pull a body of weight 50N on a rough horizontal plane is 15N. Determine the coefficient of friction if the force is applied at an angle of 15° with horizontal.

Solution



Given data

$W = 50N$

$P = 15\ N$

$\phi = 15°$

$\mu = ?$

$\Sigma H = 0$

$F = 15 \cos 15° = 14.48N$

$\Sigma V = 0$

$\Rightarrow R + 15 \sin 15° = 50$

$\Rightarrow R = 50 - 15 \sin 15°$

$= 46.11\ N.$

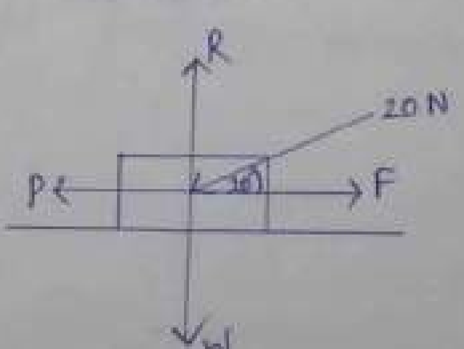
$\therefore$ coefficient of friction $(\mu) = \dfrac{F}{R}$

$= \dfrac{14.48}{46.11} = 0.314.$

Problem - 4

A body of weight 70N is placed on a rough horizontal plane. to just move the body on a horizontal plane. A push force of 20N inclined at 20° to horizontal plane is required. Find the coefficient of friction.

Solution



Given data

$W = 70N$

$P = 20N$

$\phi = 20°$

$\mu = ?$

$\Sigma H = 0$

$\Rightarrow F = 20\cos 20°$

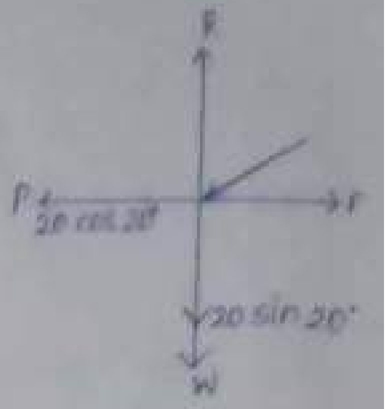
$\Rightarrow F = 18.79\,N$

$\Sigma V = 0$

$\Rightarrow R = 70 + 20\sin 20°$

$\Rightarrow R = 70 + 6.84$

$= 76.84N$



$\therefore \mu = \frac{F}{R}$

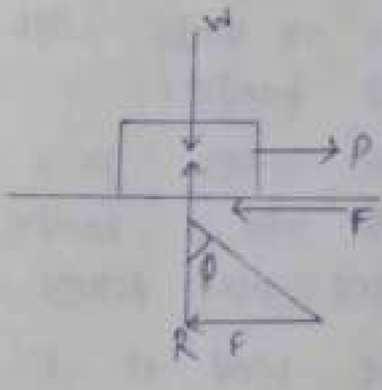
$= \frac{18.79}{76.84} = 0.244.$

Angle of friction (ϕ):-

It is defined as the angle made by resultant of the normal reaction (R) and limiting force of friction(F) with the normal reaction.

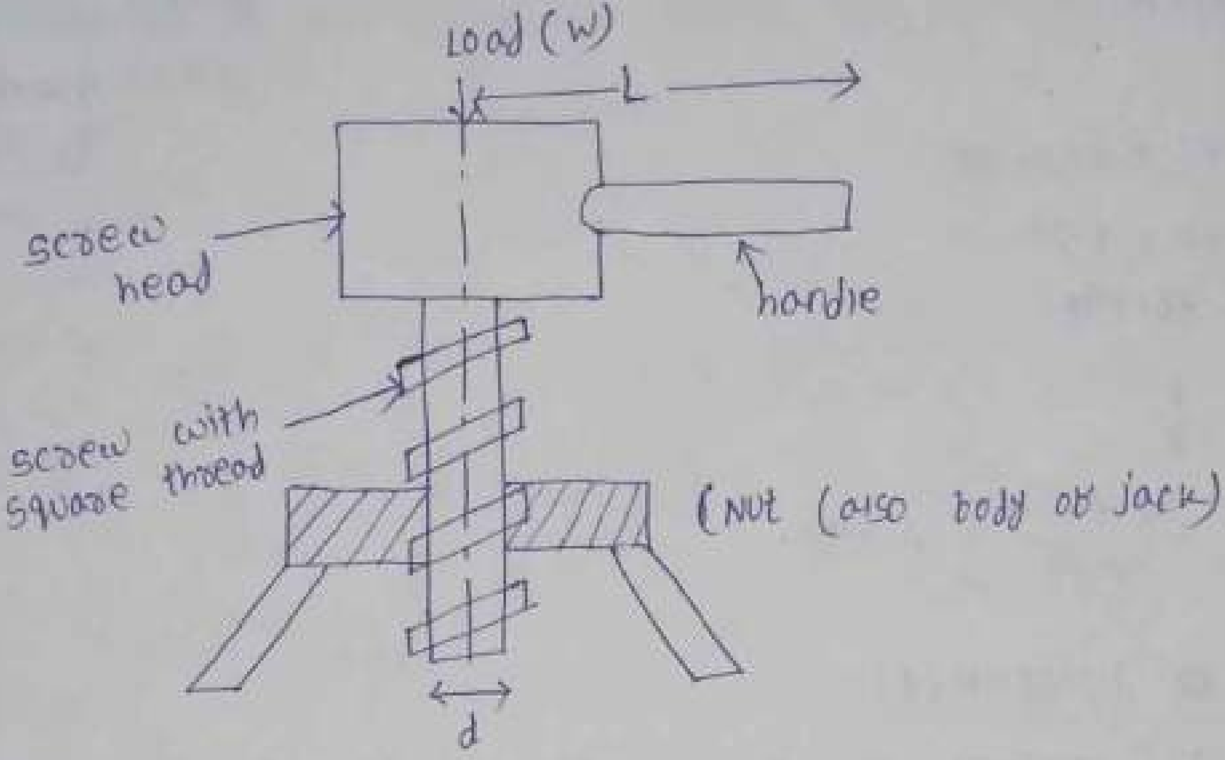
$\tan\phi = \frac{F}{R}$

$\tan\phi = \frac{\mu R}{R} = \mu.$



01.03.23

SCREW JACK



A screw jack is a device used for lifting heavy weights or loads with the help of a small effort applied at its handle.

The screw jack consist of nut, a screw with square threads and a handle fitted to the head of the screw. The nut also forms the body of jack.

The load to be lifted is placed on the top of screw. At the end of handle, fitted to the screw head, an effort 'P' is applied in horizontal direction to lift the load 'W'. The screw jack also works on the same principle on which inclined plane work.

Let, W = weight placed on the screw head.

capital P = Effort applied at the end of handle

L = length of handle

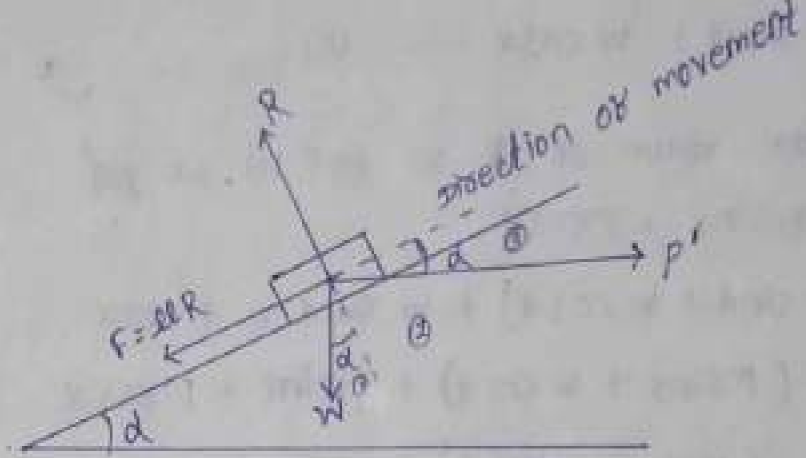
small p = pitch of screw

d = Mean diameter of screw

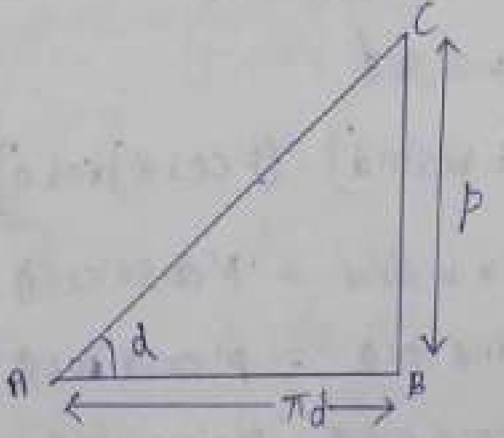
α = angle of screw / helix angle
ϕ = angle of friction
μ = co-efficient of friction between screw and nut = $\tan\phi$
$\mu = \tan\phi$.

06/03/23

Effort required at the end of handle of screw jack to lift the load W.



$1+2 = 90°$
$2+3 = 90°$
$1+2 = 2+3$
$\therefore 1 = 3 = \alpha$



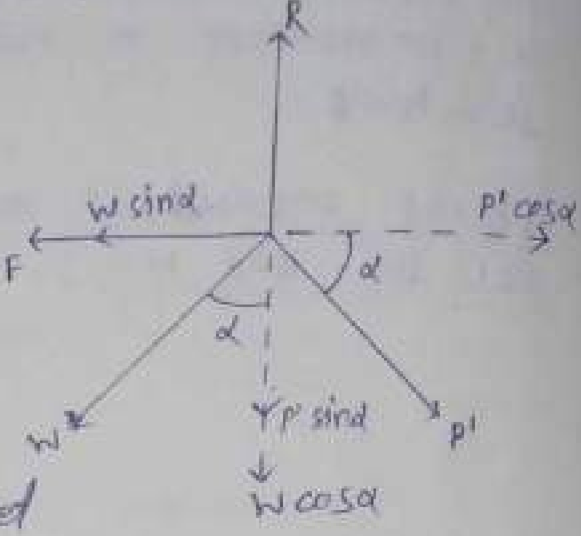
When the handle is rotate through one complete turn, the screw is also rotated through one turn. Then the load is lifted by a height (p) or pitch of the screw.

The distance AB will be equal to the circumference (πD) and the distance BC will be equal to the pitch of screw (p)

$\therefore \tan\alpha = \frac{p}{b} = \frac{p}{\pi D}$.

R = Mean radius of screw jack
P' = Effort applied horizontally at mean radius of screw jack to lift the load W.

This is similar to that of lifting a load W of an inclined plane by a horizontal force P.



$\Sigma H = 0$

$\Rightarrow F + W\sin\alpha = P'\cos\alpha$

$\Rightarrow \mu R + W\sin\alpha = P'\cos\alpha$ ---- (i)

$\Sigma V = 0$

$\Rightarrow R = P'\sin\alpha + W\cos\alpha$ ----- (ii)

putting the value of R in eqn (i), we get

$\mu R + W\sin\alpha = P'\cos\alpha$

$\Rightarrow \mu(P'\sin\alpha + W\cos\alpha) + W\sin\alpha = P'\cos\alpha$

$\Rightarrow \tan\phi\,(P'\sin\alpha + W\cos\alpha) + W\sin\alpha = P'\cos\alpha$

$\Rightarrow \frac{\sin\phi}{\cos\phi}(P'\sin\alpha + W\cos\alpha) + W\sin\alpha = P'\cos\alpha$

Multiplying both side with $\cos\phi$, we get

$\Rightarrow \cos\phi\left[\frac{\sin\phi}{\cos\phi}(P'\sin\alpha + W\cos\alpha) + W\sin\alpha\right] = (P'\cos\alpha)\cos\phi$

$\Rightarrow \sin\phi\,(P'\sin\alpha + W\cos\alpha) + \cos\phi \times W\sin\alpha = P'\cos\alpha\cos\phi$

$\Rightarrow P'\sin\alpha\sin\phi + W\cos\alpha\sin\phi + W\sin\alpha\cos\phi = P'\cos\alpha\cos\phi$

$\Rightarrow W\cos\alpha\sin\phi + W\sin\alpha\cos\phi = P'\cos\alpha\cos\phi - P'\sin\alpha\sin\phi$

$\Rightarrow W(\cos\alpha\sin\phi + \sin\alpha\cos\phi) = P'(\cos\alpha\cos\phi - \sin\alpha\sin\phi)$

$\Rightarrow W\sin(\alpha+\phi) = P'\cos(\alpha+\phi)$

$\Rightarrow P' = \frac{W\sin(\alpha+\phi)}{\cos(\alpha+\phi)}$

$\boxed{\Rightarrow P' = W\tan(\alpha+\phi)}$

$\sin(A \pm B) = \sin A\cdot\cos B \pm \cos A\cdot\sin B$

$\cos(A \pm B) = \cos A\cdot\cos B \mp \sin A\cdot\sin B$

$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\cdot\tan B}$

$\cot(A \pm B) = \frac{\cot\cdot\cot B \mp 1}{\cot B \pm \cot A}$

Now P' is the effort applied at the mean radius of screw jack. But incase of screw jack effort is actually applied at the end of handle. The effort applied at end of the handle P.

Moment of P' about the axis of screw

$M_{P'} = P' \times r = P' \times d/2$ ------ (iii)

Moment of P about the axis of screw

$M_P = P \times L$ ------ (iv)

equating eqn (iii) and (iv), we get

$\Rightarrow P' \times d/2 = P \times L$

$\Rightarrow P = \frac{P' \times d/2}{L} = \frac{P' \times d}{2L}$

$\Rightarrow P = \frac{d}{2L} \times W \tan(\alpha + \phi)$

$\Rightarrow P = \frac{d}{2L} \times W \frac{\tan\alpha + \tan\phi}{1 - \tan\alpha \times \tan\phi}$

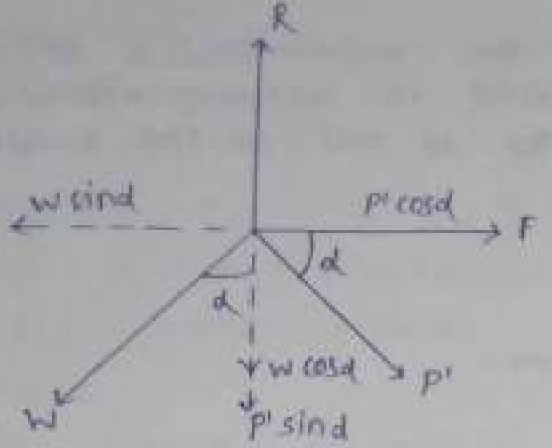
$\because \tan\alpha = \frac{p}{\pi d}$, $\tan\phi = \mu$

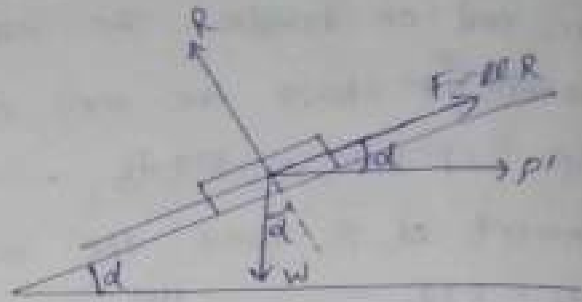
$\Rightarrow P = \frac{d}{2L} \times W \times \frac{\frac{p}{\pi d} + \mu}{1 - \frac{p}{\pi d} \times \mu}$

$\Rightarrow P = \frac{d}{2L} \times W \times \frac{\frac{p + \mu \pi d}{\cancel{\pi d}}}{\frac{\pi d - p}{\cancel{\pi d}} \times \mu}$

$\Rightarrow P = \frac{d}{2L} \times W \times \frac{p + \mu \pi d}{\pi d - p\mu}$

$$\boxed{\Rightarrow P = \frac{Wd}{2L} \times \frac{p + \mu \pi d}{\pi d - p\mu}}$$





When the handle is rotate through one complete turn, the screw is also rotated through one turn. Then the load is lifted by a height (p) or pitch of the screw.

The distance AB will be equal to the circumference (πd) and distance BC will be equal to the pitch of screw (p).

$\tan\alpha = \frac{p}{\pi d}$

r = Mean radius of screw jack

p' = effort applied horizontally at mean radius of screw jack to lift the load W.

This case is similar to that of lifting a load W of an inclined plane by a horizontal force P.

$\Sigma H = 0$

$\Rightarrow W\sin\alpha = F + P'\cos\alpha$

$\Rightarrow W\sin\alpha = \mu R + P'\cos\alpha$ ------ (i)

$\Sigma V = 0$

$\Rightarrow R = W\cos\alpha + P'\sin\alpha$ ----- (ii)

putting the value of R in eqn (i), we get

$\Rightarrow W\sin\alpha = \mu R + P'\cos\alpha$

$\Rightarrow W\sin\alpha = \mu(W\cos\alpha + P'\sin\alpha) + P'\cos\alpha$

$\Rightarrow w\sin\alpha = \tan\phi(w\cos\alpha + p'\sin\alpha) + p'\cos\alpha$

$\Rightarrow w\sin\alpha = \frac{\sin\phi}{\cos\phi}(w\cos\alpha + p'\sin\alpha) + p'\cos\alpha$

Multiply $\cos\phi$ with both side, we get

$\Rightarrow \cos\phi(w\sin\alpha) = \cos\phi\left[\frac{\sin\phi}{\cos\phi}(w\cos\alpha + p'\sin\alpha) + p'\cos\alpha\right]$

$\Rightarrow w\sin\alpha\cos\phi = \sin\phi(w\cos\alpha + p'\sin\alpha) + \cos\phi\cdot p'\cos\alpha$

$\Rightarrow w\sin\alpha\cos\phi = w\cos\alpha\sin\phi + p'\sin\alpha\sin\phi + p'\cos\alpha\cos\phi$

$\Rightarrow w\sin\alpha\cos\phi - w\cos\alpha\sin\phi = p'\sin\alpha\sin\phi + p'\cos\alpha\cos\phi$

$\Rightarrow w(\sin\alpha\cos\phi - \cos\alpha\sin\phi) = p'(\sin\alpha\sin\phi + p'\cos\alpha\cos\phi$

$\Rightarrow w\sin(\alpha-\phi) = p'\cos(\alpha-\phi)$

$\Rightarrow p' = \frac{w\sin(\alpha-\phi)}{\cos(\alpha-\phi)}$

$\boxed{\Rightarrow p' = w\tan(\alpha-\phi)}$

Now p' is the effort applied at the mean radius of screw jack. But incase of screw jack effort is actually applied at the end of handle. The effort applied at end of the handle P.

Moment of p' about the axis of screw

$M_{p'} = p' \times r = p' \times d/2$ ------ (iii)

Moment of P about the axis of screw

$M_p = P \times L$ -------- (iv)

equating eqn (iii) and (iv), we get

$\Rightarrow p' \times d/2 = P \times L$

$\Rightarrow P = \frac{p' \times d/2}{L} = \frac{p' \times d}{2L}$

$$\Rightarrow P = W \tan(\alpha - \phi) \times d/_{2L}$$

$$\Rightarrow P = \frac{Wd}{2L} \times \tan(\alpha - \phi)$$

$$\Rightarrow P = \frac{Wd}{2L} \times \frac{\tan\alpha - \tan\phi}{1 + \tan\alpha \cdot \tan\phi}$$

$$\Rightarrow P = \frac{Wd}{2L} \times \frac{\frac{p}{\pi d} - \mu}{1 + \frac{p}{\pi d} \cdot \mu}$$

$$\Rightarrow P = \frac{Wd}{2L} \times \frac{\frac{p - \mu\pi d}{\pi d}}{\frac{\pi d + p}{\pi d} \cdot \mu}$$

$$\boxed{\Rightarrow P = \frac{Wd}{2L} \times \frac{p - \mu\pi d}{\pi d + p\mu}}$$

But if $\phi > \alpha$, $P = \frac{Wd}{2L} \times \tan(\phi - \alpha)$

$$\Rightarrow P = \frac{Wd}{2L} \times \tan(\phi - \alpha)$$

$$\Rightarrow P = \frac{Wd}{2L} \times \frac{\tan\phi - \tan\alpha}{1 + \tan\phi \cdot \tan\alpha}$$

$$\Rightarrow P = \frac{Wd}{2L} \times \frac{\mu - P/_{\pi d}}{1 + \mu \cdot \frac{p}{\pi d}}$$

$$\Rightarrow P = \frac{Wd}{2L} \times \frac{\frac{\mu\pi d - p}{\pi d}}{\frac{\pi d + \mu p}{\pi d}}$$

$$\boxed{\Rightarrow P = \frac{Wd}{2L} \times \frac{\mu\pi d - p}{\pi d + \mu p}}$$

09.03.23

1. Find the effort required to applied at the end of the handle, fitted to a screw head of a screw jack to lift a load of 1500N. The length of handle is 70 cm. The mean diameter and pitch of screw jack are 6 c.m and 0.9 cm respectively. The coefficient of friction is given by 0.095. If instead of rising load of 1500N, the same load is lowered, determine the effort required to apply at the end of handle.

Ans: Given data,

Load $(W) = 1500N$

$L = 70\ cm = 0.7m$

$d = 6\ c.m = 0.06m$

$p = 0.09\ c.m = 0.009m$

$\mu = 0.095$

(i) effort required to raise the load,

$$P = \frac{Wd}{2L} \times \frac{p + \mu\pi d}{\pi d - p\mu}$$

$$= \frac{1500 \times 0.06}{2 \times 0.7} \times \frac{0.009 + 0.095 \times \pi \times 0.06}{\pi \times 0.06 - 0.009 \times 0.095}$$

$$= 9.21N.$$

(ii) effort required to lower the load,

$$P = \frac{Wd}{2L} \times \frac{\mu\pi d - p}{\pi d + p\mu}$$

$$= \frac{1500 \times 0.06}{2 \times 0.7} \times \frac{0.095 \times \pi \times 0.06 - 0.009}{\pi \times 0.06 + 0.009 \times 0.095}$$

$$= 3.02N.$$

Overhauling and self locking screw

We have seen before that $P = W\tan(\phi - \alpha)$

In the above expression if ϕ is less than α ($\phi < \alpha$) then effort required to lower the load will be negative. In other word, the load will start moving downward without application of any force. Such a condition is known as overhauling of screw.

If ϕ is greater than α ($\phi > \alpha$), the effort required to lower the load will positive indicating that an effort is applied to lower the load. Such a screw is known as self locking screw.

* In other word, a screw will be self locking if the friction angle is greater than helix angle. i.e $\phi > \alpha$.

ROLLER BEARING

Roller bearing are mechanical assemblies they are consists of cylindrical or tapper rolling elements usually captured between inner outer races.

They provide a means of supporting rotating shafts and minimize friction between the shafts and stationary machine member.

Ball bearing :-

Ball bearing are mechanical assemblies that consists of rolling sperical elements that are captured between circular inner and outer .

→ They provide a means of supporting rotating shaft and minimize friction between the shafts and stationary machine member.

→ Ball bearings are used primarily in machinary that has shafts requiring support for low friction rotation.

→ Ball bearings are also known as rolling element bearing or anti friction bearing.

NEEDLE ROLLER BEARING:-

→ Needle roller bearing are a spherical style of bearing that cylindrical rollers with small diameters that offer a higher load carrying capacity.

→ Then also can withstand with greater regidity throw other options.

Friction of pivot and collar bearing

→ The rotating shaft are frequently subjected to axial thrust the bearing surfaces such as pivot and collar bearing are used to take this axial thrusts, of the rotating shaft.

→ Bearing surface placed at the end of the shaft to take the axial thrusts are known as pivots. The pivot may have a flat surface or conical surface.

→ When the

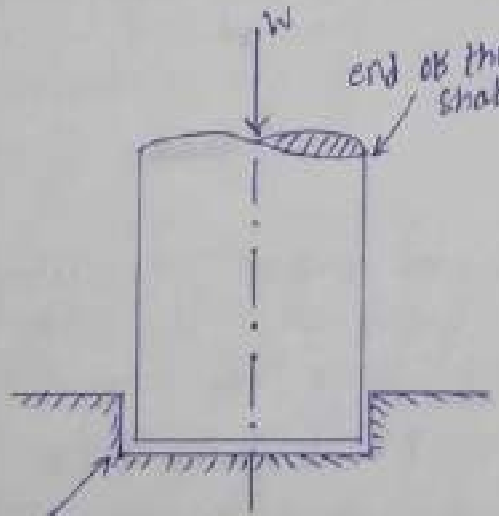
→ The collar bearing may have flat surface or conical bearing surface, but the flat surface is most commonly used. They are may be single collar or several collar along the length of the shaft in order to reduced to intensity of pressure

A little consideration will show that in a new bearing, the contact between the shaft and bearing may be good over the whole surface. In other words, we can say that the pressure over the rubbing surface is uniformly distributed. But when the bearing becomes old, all parts of rubbing surface will not move with the same viscocity. because the velocity of rubbing surface increase with the distance from the axis bearing. This means that wear, may be different at different raddi and this cause to alter the distribution of pressure, hence in the study of friction of bearing, it is assumed that

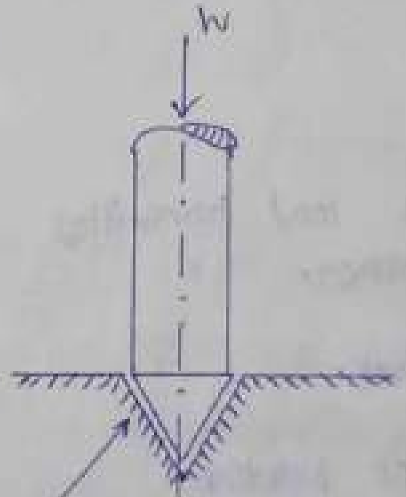
11.03.22

Pivot and collar bearing

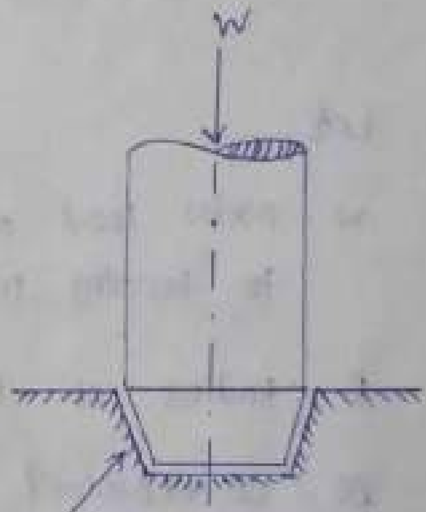
- The rotating shafts are frequently subjected to an axial thrust. These shafts can be in correct axial position if bearing surfaces are provided.
- The bearing surfaces place at the end of a shaft are known as pivots.
- The bearing surfaces provided at any position along with the shaft but not at the end of the shaft to carry the axial thrust is known as collar.
- The pivot may have a flat surfaces or a conical surfaces or truncated.
- The surface of collar may be flat or conical shape.



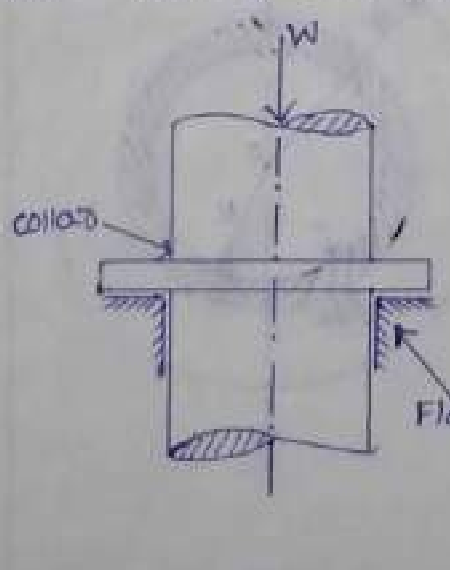
Flat bearing surfaces



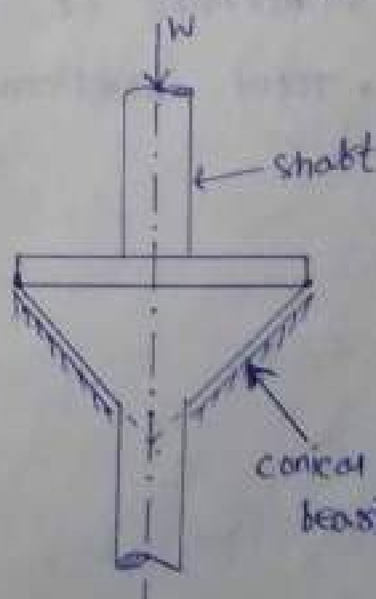
conical bearing surface



Turning bearing surface



Flat collar bearing



conical car bearing

The design of bearings is based on the following assumptions :-

(i) The pressure is uniformly distributed over the bearing surface.

(ii) The wear is uniform over the bearing surface.

Flat pivot

→ The bearing surface placed at the end of shaft is known as pivot. If the bearing surface is flat then the bearing surface is called flat pivot or foot step.

→ There will be friction along the surface of contact between shaft and bearing.

Let,

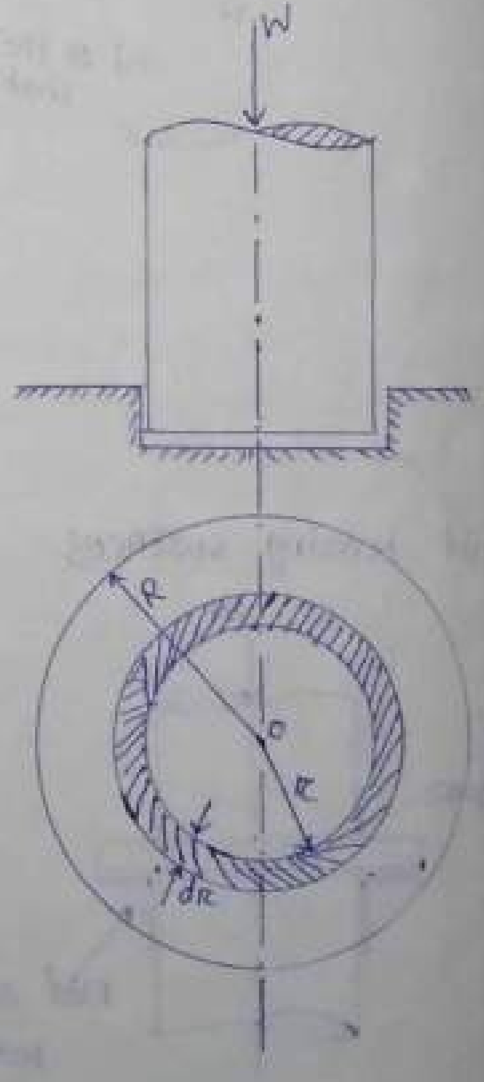
W = axial load or load transmitted to bearing surface.

R = Radius of pivot.

μ = co-efficient of friction.

P = intensity of pressure in N/m^2

T = Total frictional torque



consider a circular ring of radius 'r' and thickness 'dr' as shown in figure.

Area of ring = $2\pi r \times dr$

case-1 (uniform pressure)

When pressure is assumed to be uniform over the bearing surface, then intensity of pressure is given by

$$p = \frac{\text{axial load}}{\text{area of cross section}} = \frac{W}{\pi R^2}$$

pressure on ring, $p = \frac{W}{2\pi r \times dr}$

$\Rightarrow W = p \times 2\pi r\, dr$ (load transmitted to the ring)

Frictional force on ring, $(F) = \mu \times$ load on ring

$= \mu \times W$

$= \mu \times p \times 2\pi r\, dr$

Frictional torque on ring (dT) = Frictional force $\times$ radius

$= F \times r$

$= \mu \times p \times 2\pi r\, dr \times r$

$= 2\pi \mu p r^2 dr$

Total frictional torque $(T) = \int_0^R dT$

$$= \int_0^R 2\pi \mu p r^2 dr$$

$$= 2\pi \mu p \int_0^R r^2 dr$$

$$= 2\pi \mu p \left[\frac{R^3}{3}\right]_0^R$$

$$\Rightarrow T = 2\pi \mu p \frac{R^3}{3}$$

by putting value p we get.

$$T = 2\pi \mu p \frac{R^3}{3}$$

$$= 2\pi \mu \times \frac{W}{\pi R^2} \times \frac{R^3}{3}$$

$$= \frac{2}{3} \mu W R \text{ N·m}$$

Power lossed in friction

$$P = T \times \omega$$

$$= T \times \frac{2\pi N}{60}$$

$$= \frac{2\pi N T}{60}$$

Case-II (For uniform wear) 16.03.23

For the uniform wear of bearing surface, the load transmitted to various circular ring should be same. But the load transmitted to any circular ring is equal to the product of pressure and area of ring.

For uniform wear the product of pressure and area of ring should be constant. Area of ring is directly proportional to radius of ring. Hence for uniform wear, the product of pressure and radius should be constant.

$$p \times r = \text{constant}$$

$$\therefore p = \frac{C}{r}$$

Load transmitted to ring = pressure × Area of ring

$$= p \times 2\pi r dr$$

putting the value of p above, we get.

$= p \times 2\pi r\, dr$

$= \frac{c}{r} \times 2\pi r\, dr$

$= 2\pi c\, dr$.

Total load transmitted to bearing

$= \int_0^R 2\pi c\, dr$

$= 2\pi c \int_0^R dr$

$\Rightarrow W = 2\pi c R$.

$\therefore C = \frac{W}{2\pi R}$.

Frictional force on ring (F) = $\mu \times$ load on ring

$= \mu \times 2\pi c\, dr$

Frictional torque on ring (dT) = frictional force $\times$ radius

$= \mu \times 2\pi c\, dr \times r$

$= 2\pi \mu c\, r\, dr$

Total frictional torque on bearing $= \int_0^R dT$

$= \int_0^R 2\pi\mu c\, r\, dr$

$T = 2\pi\mu c \int_0^R r\, dr$

$= 2\pi\mu c \left[\frac{r^2}{2}\right]_0^R$

$= 2\pi\mu c \frac{R^2}{2}$

$T = \pi\mu c R^2$.

By putting the value 'c' above, we get

$= \pi \mu c R^2$

$= \pi \mu \frac{W}{2\pi R} R^2$

$= \frac{\mu W R}{2}$.

Power lossed due to friction

$P = T \times \omega$

$= T \times \frac{2\pi N}{60}$

$= \frac{2\pi N T}{60}$ watt.

Problem

Find the power lost in friction assuming
(i) uniform pressure
(ii) uniform wear
when a vertical shaft of 100 mm diameter rotating 150 rpm rests on a flat end foot step bearing. The coefficient of friction is equal to 0.05 and shaft carries a vertical load of 15 KN.

Ans: Given data,

$d = 100mm = 0.1m$

$N = 150\ rpm$

$\mu = 0.05$

$W = 15\ KN = 15 \times 10^3\ N$

$R = \frac{0.1}{2} = 0.050m$

Ans: Uniform pressure

$T = \frac{2}{3} \mu WR$

$= \frac{2}{3} \times 0.05 \times 15 \times 10^3 \times 0.05$

$= 25 \text{ Nm}$

Power lost in friction $(P) = \frac{2\pi NT}{60}$

$= \frac{2\pi \times 150 \times 25}{60} = 392.69$ watt.

Uniform wear

$T = \frac{1}{2} \mu WR$

$= \frac{1}{2} \times 0.05 \times 15 \times 10^3 \times 0.05$

$= 18.75 \text{ Nm}$

$P = \frac{2\pi NT}{60}$

$= \frac{2\pi \times 150 \times 18.75}{60} = 294.52 \text{ W}$

Flat collar bearing

The bearing surface provided at any position along the shaft, (but not at the end of shaft), to carry axial thrust is known as collar. which may be flat or conical. If the surface is flat then the bearing surface is known as flat bearing.

The collar bearing also known as thrust bearing.

consider a collar bearing as shown in figure.

Let,

W = axial load or load transmitted to bearing surface

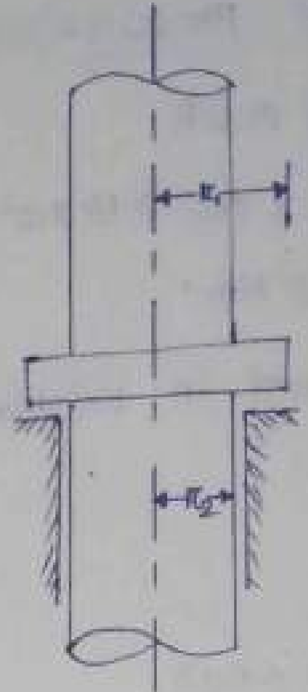
R_1 = External radius of the collar.

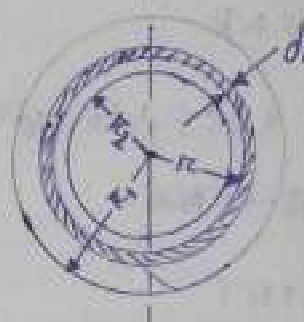
R_2 = Internal radius of the collar.

μ = co-efficient of friction

P = Intensity of pressure

T = Total frictional torque.





consider a circular ring of radius 'R' and thickness 'dR' as shown in figure.

Area of ring = $2\pi R \times dR$

Area of the bearing surface = $\pi[(R_1)^2 - (R_2)^2]$

<u>case -1</u> (uniform pressure)

when the pressure is uniformly distributed over the bearing surface, then the intensity of pressure,

$$P_b = \frac{\text{axial load}}{\text{area of bearing surface}}$$

$$= \frac{W}{A} = \frac{W}{\pi[(R_1)^2 - (R_2)^2]}$$

intensity pressure on ring, $(P_r) = \frac{\text{Load on ring}}{\text{area of ring}}$

$$P = \frac{W}{2\pi R \times dR}$$

$\therefore W = P \times 2\pi R \times dR$

Frictional force on ring $(F) = \mu \times$ load on ring

$$= \mu \times P \times 2\pi R \times dR$$

$$= 2\pi \mu P R dR$$

Frictional torque on ring (dT) = Frictional force $\times$ radius

$$= 2\pi \mu P R dR \times R$$

$$= 2\pi \mu P R^2 dR$$

Intigrating this equation within the limits from R_2 to R_1 for the total frictional torque on the collar.

$\therefore$ Total frictional torque $(T) = \int_{R_2}^{R_1} dT$

$$T = \int_{R_2}^{R_1} 2\pi \mu P R^2 dR$$

$$= 2\pi \mu P \int_{R_2}^{R_1} R^2 dR$$

$$= 2\pi \mu P \left[\frac{R^3}{3}\right]_{R_2}^{R_1}$$

$$= 2\pi \mu P \left[\frac{(R_1)^3}{3} - \frac{(R_2)^3}{3}\right]$$

$$= 2\pi \mu P \left[\frac{(R_1)^3 - (R_2)^3}{3}\right] \quad \text{----- (i)}$$

Substituting the value of 'p' in eqⁿ (i), we get

$$T = 2\pi \mu p \left[\frac{(R_1)^3 - (R_2)^3}{3} \right]$$

$$= 2\cancel{\pi}\mu \times \frac{W}{\cancel{\pi}\left[(R_1)^2 - (R_2)^2\right]} \left[\frac{(R_1)^3 - (R_2)^3}{3} \right]$$

$$= \frac{2}{3} \mu W \left[\frac{(R_1)^3 - (R_2)^3}{(R_1)^2 - (R_2)^2} \right]$$

Power lossed due to friction

$$P = T \times \omega$$

$$= T \times \frac{2\pi N}{60}$$

$$= \frac{2\pi N T}{60}$$

Case-II (uniform wear)

For the uniform wear of bearing surface, the load transmitted to various circular ring should be same. But the load transmitted to any circular ring is equal to the product of pressure and area of ring.

For uniform wear the product of pressure and area of ring should be constant. Area of ring is directly proportional to the radius of ring. Hence for uniform wear the product of pressure and radius should be constant.

$$P \times r = \text{constant}$$

$$\therefore P = \frac{C}{r}$$

Load transmitted to ring (W) = pressure x area of ring

$= P \times 2\pi r dr$ ......(1)

putting the value of P in eqn (1), we get

$= P \times 2\pi r dr$

$= \frac{C}{\cancel{r}} \times 2\pi \cancel{r} dr$

$= 2\pi C dr.$

Total load transmitted to the bearing or collar

$\Rightarrow W = \int_{r_2}^{r_1} 2\pi C dr$

$= 2\pi C \int_{r_2}^{r_1} dr$

$= 2\pi C [r]_{r_2}^{r_1}$

$\Rightarrow W = 2\pi C (r_1 - r_2)$

$\Rightarrow C = \frac{W}{2\pi (r_1 - r_2)}$

Frictional force on ring ring (F) = μ x load on ring

$= \mu \times 2\pi C dr$

Frictional torque on ring (dT) = Frictional force x radius

$= 2\pi \mu C dr \times r$

$= 2\pi \mu C r dr$

Total frictional torque on the bearing,

$T = \int_{r_2}^{r_1} 2\pi \mu C r dr$

$$T = 2\pi \omega C \int_{R_2}^{R_1} R \, dR$$

$$= 2\pi \omega C \left[\frac{R^2}{2}\right]_{R_2}^{R_1}$$

$$= 2\pi \omega C \left[\frac{(R_1)^2 - (R_2)^2}{2}\right]$$

$$= \pi \omega C \left[(R_1)^2 - (R_2)^2\right] \text{ ----- (ii)}$$

substituting the value of 'C' in eqn (ii), we get

$$T = \pi \omega C \left[(R_1)^2 - (R_2)^2\right]$$

$$= \pi \omega \times \frac{W}{2\pi(R_1 - R_2)} \times \left[(R_1)^2 - (R_2)^2\right]$$

$$= \frac{1}{2} \omega W \times \frac{(R_1)^2 - (R_2)^2}{(R_1 - R_2)}$$

$$= \frac{1}{2} \omega W \times \frac{(R_1 + R_2)\cancel{(R_1 - R_2)}}{\cancel{(R_1 - R_2)}}$$

$$= \frac{1}{2} \omega W \times (R_1 + R_2).$$

power lossed due to friction,

$$P = T \times \omega$$

$$= T \times \frac{2\pi N}{60}$$

$$= \frac{2\pi N T}{60}.$$

① problem

In a collar thrust bearing the external and internal radii are 250mm and 150mm respectively. The total axial load is 50kN and shaft is rotating at 150 rpm. The co-efficient of friction is 0.05. Find the power lossed in friction assuming uniform pressure and uniform wear.

solution

Given data,

$r_1 = 250\,mm = 0.25\,m$

$r_2 = 150\,mm = 0.15\,m$

$W = 50\,kN = 50 \times 10^3\,N$

$N = 150\,rpm$

$\mu = 0.05$

For uniform pressure:-

$$T = \frac{2}{3}\mu W \times \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$$

$$= \frac{2}{3} \times 0.05 \times 50 \times 10^3 \left[\frac{(0.25)^3 - (0.15)^3}{(0.25)^2 - (0.15)^2}\right]$$

$$= 510.41\,Nm.$$

$$P = \frac{2\pi N T}{60}$$

$$= \frac{2\pi \times 150 \times 510.41}{60} = 8017.50\,W.$$

For uniform wear

$$T = \frac{1}{2}\mu W (r_1 + r_2)$$

$$= \frac{1}{2} \times 0.05 \times 50 \times 10^3 (0.25 + 0.15) = 500\,Nm.$$

$$P = \frac{2\pi NT}{60}$$

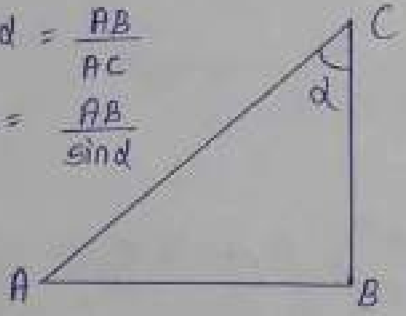
$$= \frac{2\pi \times 150 \times 500}{60} = 7853.98\ W.$$

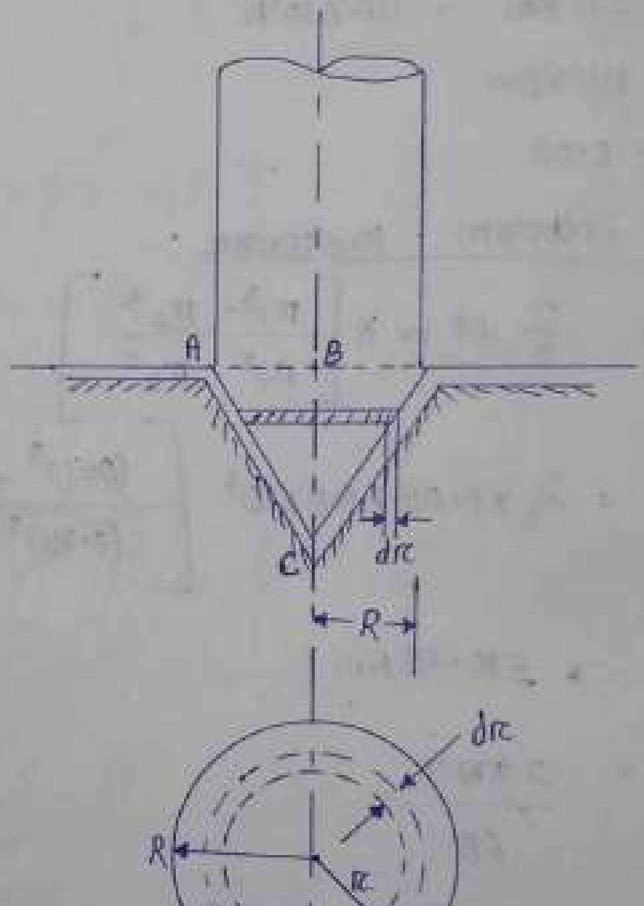
Conical pivot bearing

The bearing surface placed at the end of a shaft and having a conical surface is known as conical pivot bearing.

$$\sin\alpha = \frac{AB}{AC}$$

$$\Rightarrow AC = \frac{AB}{\sin\alpha}$$





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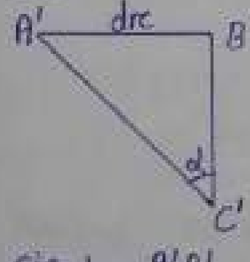


$$\sin\alpha = \frac{A'B'}{A'C'}$$

$$A'C' = \frac{A'B'}{\sin\alpha} = \frac{dr}{\sin\alpha}$$

$(\because A'B' = A'C' = dr)$

Let,

W = axial load or load transmitted to bearing surface

μ = coefficient of friction

R = Radius of bearing

α = semiangle of cone

p = pressure intensity normal to the cone surface

consider a circular ring of radius 'r' and thickness 'dr'.

Actual thickness of sloping ring $= \frac{dr}{\sin\alpha}$

$\because$ sloping length of ring $\Rightarrow A'C' = \frac{A'B'}{\sin\alpha}$

$= \frac{dr}{\sin\alpha}$

Area of ring along conical surface = $2\pi r$ x actual thickness of sloping ring

$= 2\pi r \times \frac{dr}{\sin\alpha}$

Case-1 (uniform pressure)

when the pressure is uniformly distributed over bearing surface, then the intensity pressure,

$$p = \frac{W}{\pi R^2}$$

Pressure on the ring = $\dfrac{\text{load on the ring}}{\text{area of ring along conical surface}}$

$$p = \frac{W}{2\pi r \times \frac{dr}{\sin\alpha}}$$

Load on the ring normal to conical surface

$$\Rightarrow W = P \times 2\pi r \frac{dr}{\sin\alpha}$$

vertical component of above load $= P \times 2\pi r \frac{dr}{\cancel{\sin\alpha}} \times \cancel{\sin\alpha}$

$$= P \times 2\pi r \, dr$$

Total vertical load transmitted to bearing (W)

$$\Rightarrow W = \int_0^R P \times 2\pi r \, dr$$

$$= P \times 2\pi \int_0^R r \, dr$$

$$= P \times 2\pi \left[\frac{r^2}{2}\right]_0^R$$

$$= P \times \cancel{2}\pi \times \frac{R^2}{\cancel{2}}$$

$$\Rightarrow W = P \times \pi \times R^2 .$$

$$\Rightarrow P = \frac{W}{\pi R^2}$$

Frictional force on the ring along conical surface

$= \mu \times$ load on ring normal to conical surface

$$= \mu \times P \times 2\pi r \frac{dr}{\sin\alpha}$$

Frictional torque on ring = Frictional force x radius

$$dT = \mu \times P \times 2\pi r \frac{dr}{\sin\alpha} \times r$$

$$= \mu \times P \times 2\pi r^2 \frac{dr}{\sin\alpha}$$

Total frictional torque on bearing (T) = $\int_0^R dT$

$$\Rightarrow T = \int_0^R \mu \times p \times 2\pi r^2 \frac{dr}{\sin\alpha}$$

$$= \frac{\mu \times 2\pi p}{\sin\alpha} \int_0^R r^2\, dr$$

$$= \frac{\mu \times 2\pi p}{\sin\alpha} \left[\frac{r^3}{3}\right]_0^R$$

$$= \frac{\mu \times 2\pi p}{\sin\alpha} \times \frac{R^3}{3} \quad \cdots \text{(i)}$$

Putting the value 'p' in eqn (i), we get

$$= \frac{\mu \times 2\pi p}{\sin\alpha} \times \frac{R^3}{3}$$

$$= \frac{\mu \times 2\cancel{\pi}}{\sin\alpha} \times \frac{W}{\cancel{\pi} R^2} \times \frac{R^{\cancel{3}}}{3}$$

$$= \frac{2}{3} \frac{\mu W R}{\sin\alpha}.$$

power lossed due to friction

$$P = T \times \omega$$

$$= T \times \frac{2\pi N}{60}$$

$$= \frac{2\pi N T}{60}.$$

Case - II (uniform wear)

We know for uniform wear product of pressure and radius is constant.

$P \times r = C$

$P = \frac{C}{r}$.

We know that normal load acting on the ring

$$\Rightarrow W_n = \text{Normal Pressure} \times \text{Area}$$

$$= P_n \times 2\pi r \frac{dr}{\sin\alpha}$$

vertical load acting on the ring

$$\Rightarrow W = W_n \times \sin\alpha$$

$$= P_n \times 2\pi r \frac{dr}{\cancel{\sin\alpha}} \times \cancel{\sin\alpha}$$

$$= P_n \times 2\pi r\, dr$$

Total vertical load transmitted to bearing

$$= \int_0^R \text{Load on ring}$$

$$= \int_0^R P \times 2\pi r\, dr \quad = \int_0^R \frac{C}{\cancel{r}} \times 2\pi \cancel{r}\, dr$$

$$= C\, 2\pi \int_0^R dr$$

$$= C \times 2\pi \left[R \right]_0^R$$

$$\Rightarrow W = 2\pi C R$$

$$\Rightarrow C = \frac{W}{2\pi R}$$

Frictional torque on ring (dT) = Frictional load on ring $\times$ radius

$$= \mu \times p \times 2\pi r \frac{dr}{\sin\alpha} \times r$$

$$= \mu \times \frac{C}{r} \times 2\pi r \frac{dr}{\sin\alpha} \times r$$

$$= \mu \times C \times 2\pi r \frac{dr}{\sin\alpha} \quad \text{---- (i)}$$

putting the value 'C' in eqn(i), we get

$$= \mu \times C \times 2\pi r \frac{dr}{\sin\alpha}$$

$$= \mu \times \frac{W}{2\pi R} \times 2\pi r \frac{dr}{\sin\alpha}$$

$$= \mu \times \frac{W}{R} \times r \frac{dr}{\sin\alpha}$$

Total frictional torque $(T) = \int_0^R dT$

$$= \int_0^R \mu \times \frac{W}{R} \times r \frac{dr}{\sin\alpha}$$

$$= \frac{\mu \times W}{R \times \sin\alpha} \int_0^R r\,dr$$

$$= \frac{\mu \times W}{R \times \sin\alpha} \left[\frac{r^2}{2}\right]_0^R$$

$$= \frac{\mu \times W}{R \times \sin\alpha} \times \frac{R^2}{2}$$

$$= \frac{1}{2} \frac{\mu W R}{\sin\alpha}$$

Power lossed due to briction

$$P = T \times \omega$$

$$= T \times \frac{2\pi N}{60}$$

$$= \frac{2\pi NT}{60}.$$

Problem

A conical pivot with angle of cone as 120° support a vertical shaft of diameter 300 mm. It is subjected to a load of 20 kN. The coefficient of briction is 0.05 and the speed of shaft is 200 rpm. calculate the power lossed in briction assuming (i) uniform pressure
(ii) uniform wear

Solution

Given data.

$2\alpha = 120°$, $\alpha = 60°$

$d = 300\ mm = 0.3 m$

$R = \frac{d}{2} = \frac{0.3}{2} = 0.15\ m$

$W = 20\ kN = 20 \times 10^3 N$

$\mu = 0.05$

$N = 200\ rpm$

$P = ?$

(i) uniform pressure :-

$$T = \frac{2}{3} \frac{\mu W R}{\sin\alpha}$$

$$= \frac{2}{3} \times \frac{0.05 \times 20 \times 10^2 \times 0.15}{\sin 60} = 115.47 Nm.$$

$$P = \frac{2\pi NT}{60}$$

$$= \frac{2\pi \times 200 \times 115.47}{60}$$

$= 2418.39$ watt

$= 2.41\ KW$

(ii) uniform wear :-

$$T = \frac{1}{2}\frac{\mu WR}{\sin\alpha}$$

$$= \frac{1}{2} \times \frac{0.05 \times 20 \times 10^3 \times 0.15}{\sin 60}$$

$= 86.60\ Nm$

$$P = \frac{2\pi NT}{60}$$

$$= \frac{2\pi \times 200 \times 86.60}{60}$$

$= 1813.74\ W$

$= 1.81\ KW.$

22.03.23

Friction clutch

The device used to transmit the rotary motion of one shaft to another, the axis of which are coincident, is known as clutch or friction clutch. With the help of friction clutch, the power is transmitted from one shaft to another shaft which much be started or stoped frequently.

In case of automobile clutch is used to transmit the power from engine to gear box. Through gear box we engage or disengage the gear box with engine to change the gear.

Following types of friction clutch are mostly used :-

(i) Disk clutch or single plate clutch
(ii) Multi plate clutch
(iii) cone clutch

The clutch plate is positioned between the flywheel and a solid plate is known as pressure plate.

The principle of disc and cone clutch are same as that of pivot and collar bearings.

Through cone clutch and ~~multiple~~ disc clutch are no longer in used for power transmission of directly from engine shaft by solid friction, multiple plate clutch is mostly used in automobiles.

All modern cars having single plate clutch with a fabric facing on each side of plate.

(i) Disc clutch or single plate clutch:-

Let, r_1 = external radius of friction lining on clutch plate.

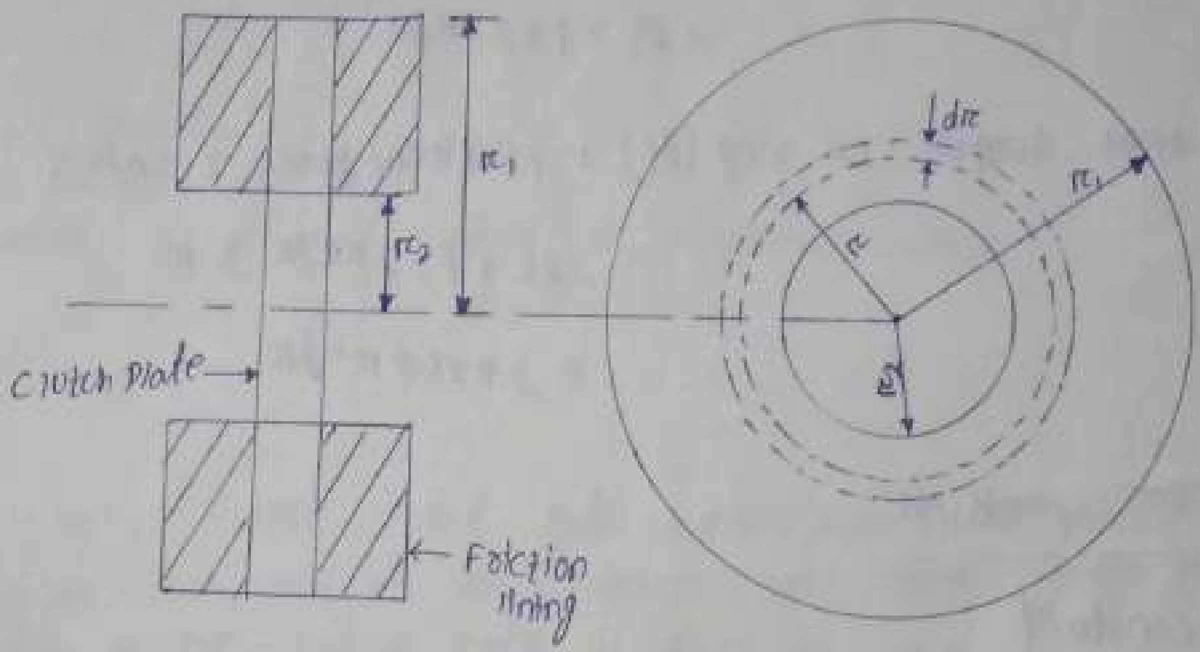
r_2 = Internal radius of friction lining on clutch plate.

p = Intensity of pressure

μ = coefficient of friction

W = Total axial load

T = Torque transmitted



The theory of single plate clutch is also based on the same principle as that of collar bearing. In case of collar bearing the power lossed due to friction should be reduced and hence the value of coefficient of friction should be decrease.

But in case of clutch the power transmitted by friction lining should be more and hence co-efficient of friction should be increase.

consider a circular ring of radius 'r' and thickness 'dr' as shown in figure.

Area of ring $(dA) = 2\pi r \times dr$

axial load on ring = pressure x area of ring

$= p \times 2\pi r \times dr$

Frictional force on ring $(dF) = \mu \times$ load on ring

$= \mu \times p \times 2\pi r dr$

Frictional torque on ring (dT) = Frictional force x radius

$= \mu \times p \times 2\pi r dr \times r$

$= 2\pi \mu p r^2 dr.$

Uniform pressure

$p =$ constant

$$p = \frac{\text{Force}}{\text{Area}} = \frac{W}{\pi r_1^2 - \pi r_2^2} = \frac{W}{\pi\left((r_1)^2 - (r_2)^2\right)}$$

Total frictional torque acting on friction surface

$$T = \int_{r_2}^{r_1} dT$$

$$= \int_{r_2}^{r_1} 2\pi \mu p r^2 dr$$

$$= 2\pi \mu p \int_{r_2}^{r_1} r^2 dr$$

$$= 2\pi \mu p \left[\frac{r^3}{3}\right]_{r_2}^{r_1}$$

$$= 2\pi \mu \times \frac{W}{\pi(r_1^2 - r_2^2)} \left[\frac{r_1^3 - r_2^3}{3}\right]$$

$$= \frac{2}{3} \mu W \times \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$$

$$= \mu W \frac{2}{3} \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$$

$$= \mu W R_m$$

where, R_m = mean radius of friction surface

$$= \frac{2}{3} \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$$

In a single clutch plate, there are two friction surface one on each side of the friction plate, hence total frictional torque on the clutch plate is given by,

$$T^* = 2T$$

$$T^* = 2 \times \frac{2}{3} \mu W \times \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$$

Uniform wear

$p \times r$ = constant

$$p = \frac{c}{r}$$

Total load on ring (dw) = press × Area of ring

$$= p \times 2\pi r \times dr$$

Total axial load (W) $= \int_{r_2}^{r_1} dW$

$$= \int_{r_2}^{r_1} p \times 2\pi r\, dr$$

$$= \int_{r_2}^{r_1} \frac{C}{r} \times 2\pi r\, dr$$

$$= \int_{r_2}^{r_1} 2\pi C\, dr$$

$$= 2\pi C \int_{r_2}^{r_1} dr$$

$$= 2\pi C \left[r \right]_{r_2}^{r_1}$$

$$W = 2\pi C (r_1 - r_2)$$

$$\Rightarrow C = \frac{W}{2\pi (r_1 - r_2)}$$

Frictional force on ring (dF) $= \mu \times$ load on ring

$$= \mu \times p \times 2\pi r\, dr$$

Frictional torque on ring (dT) $= dF \times$ radius

$$= \mu \times p \times 2\pi r\, dr \times r$$

$$= \mu \times \frac{C}{r} \times 2\pi r\, dr \times r$$

$$= \mu \times C \times 2\pi r\, dr$$

Total frictional torque (T) = $\int_{r_2}^{r_1} dT$

$$= \int_{r_2}^{r_1} \mu \times C \times 2\pi r\,dr$$

$$= 2\pi\mu C \int_{r_2}^{r_1} r\,dr$$

$$= 2\pi\mu C \left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

$$= 2\pi\mu C \left[\frac{r_1^2 - r_2^2}{2}\right]$$

$$= 2\pi\mu \times \frac{W}{2\pi(r_1 - r_2)} \times \frac{r_1^2 - r_2^2}{2}$$

$$= \cancel{2\pi}\mu \times \frac{W}{\cancel{2\pi(r_1-r_2)}} \times \frac{(r_1+r_2)\cancel{(r_1-r_2)}}{2}$$

$$T = \frac{1}{2}\mu W(r_1+r_2)$$

$$T = \mu W R_m$$

where, R_m = mean radius $= \frac{(r_1+r_2)}{2}$

Total torque on single plate clutch is given by,

$$T^* = 2T$$

$$\Rightarrow T^* = \cancel{2} \times \frac{1}{\cancel{2}}\mu W(r_1+r_2)$$

$$= \mu W(r_1+r_2)$$

NOTE :-
For power transmission by friction through a clutch uniform wear theory gives safer result. Hence uniform wear should be assumed in case of friction clutch, unless it is specified otherwise.

Multi plate clutch

Multi plate clutch is used when a large torque is to be transmitted such as incase of motor cars and machine tool.

r_1 = External radius of friction lining on friction plate.
r_2 = Internal radius of friction lining on friction plate.
W = Total axial load
p = Intensity of pressure
n_1 = no. of friction plates on driving shafts ~~to~~
n_2 = no. of friction plates on driven shaft.
n = no. of active surface or friction surface.

$$\boxed{n = n_1 + n_2 - 1}$$

Total torque transmitted is given by;

$$T = n \mu W R_m$$

where, R_m = mean radius

$$R_m = \frac{r_1 + r_2}{2} \quad \text{(uniform wear)}$$

$$R_m = \frac{2}{3}\left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right) \quad \text{(uniform pressure)}$$

Q calculate the power transmitted by a single plate clutch at a speed of 2000 rpm. If the outer and inner radii of friction surface are 150 mm and 100 mm respectively. The maximum intensity of pressure at any point of contact surface should not exceed 0.8×10^5 N/m². Take both side of the plate are effective and coefficient of friction = 0.3.
Assume uniform wear.

Ans: Given data,

$N = 2000$ rpm

$r_1 = 150$ mm $= 0.15$ m

$r_2 = 100$ mm $= 0.1$ m

No. of effective side = 2

$P_{max} = 0.8 \times 10^5$ N/m²

$\mu = 0.3$

We know at uniform wear

$p \times r = C$

$\Rightarrow (0.8 \times 10^5) \times 0.1 = C$

$\Rightarrow C = 8000$.

We know axial load (W) $= 2\pi C (r_1 - r_2)$

$= 2\pi \times 8000 (0.15 - 0.1)$

$= 2513.27$ N.

Torque due to both side active surface,

$$\Rightarrow T^* = 2 \times \left[\frac{1}{2} \mu W (r_1 + r_2) \right]$$

$$= 0.3 \times 2513.27 (0.15 + 0.1)$$

$$= 188.49 \text{ Nm}.$$

Power transmitted (P) $= \frac{2\pi N T^*}{60}$

$$= \frac{2\pi \times 2000 \times 188.49}{60} = 39477.25 \text{ W}.$$

$$= 39.47 \text{ kW}.$$

② A multi-clutch has six plates (friction rings) on the driving shaft and six plates on driven shaft. The external radius of friction surface is 115mm where as internal radius 80mm. Assuming uniform wear and coefficient of friction as 0.1, find the power transmitted at 2000rpm. Axial intensity of pressure is not to exceed 0.16 N/mm²

Ans: Given data,

$n_1 = 6$

$n_2 = 6$

$n = n_1 + n_2 - 1$

$= 6 + 6 - 1 = 11$

$r_1 = \cancel{80mm}\ 115mm = 0.115m$

$r_2 = 80mm = 0.08m$

$\mu = 0.1$

$P = ?$

$N = 2000$ rpm

$p = 0.16\ N/mm^2 = 0.16 \times 10^6\ N/m^2$

$T = n \mu W R_m$.

$\therefore R_m = \frac{r_1 + r_2}{2} = \frac{0.115 + 0.08}{2} = 0.0975\ m$

at uniform wear, $pr = C$

$\Rightarrow 0.16 \times 10^6 \times 0.08 = C$

$\Rightarrow C = 12800$.

axial load $(W) = 2\pi C (r_1 - r_2)$

$= 2\pi \times 12800 (0.115 - 0.08)$

$= 2814.86\ N$.

$T = n\mu \omega R_m$

$= 11 \times 0.1 \times 2814.86 \times 0.0975$

$= 301.89 \, Nm.$

power Transmitted (P) $= \dfrac{2\pi NT}{60}$

$= \dfrac{2\pi \times 2000 \times 301.89}{60}$

$= 63227.69 \, W$

$= 63.227 \, kW.$

24.03.23

BRAKE

A brake is a device used either to bring a body to rest which is in motion or to hold a body in a state of rest or of uniform motion against the action of external force or couples.

Simple frictional brake:-

A brake is a device by means of which artificial frictional resistance is applied to a moving machine member, in order to retard or stop the motion of a machine.

In the process of performing this function, the brakes absorbs either kinetic energy of moving member or potential energy.

The energy absorbed by brakes is dissapointed in the form of heat. The heat is disappiated in the surrounding there, so the excessive of brake lining does not takes place.

* The measure functional difference between the clutch and brake is, that a clutch is used to keep the driving

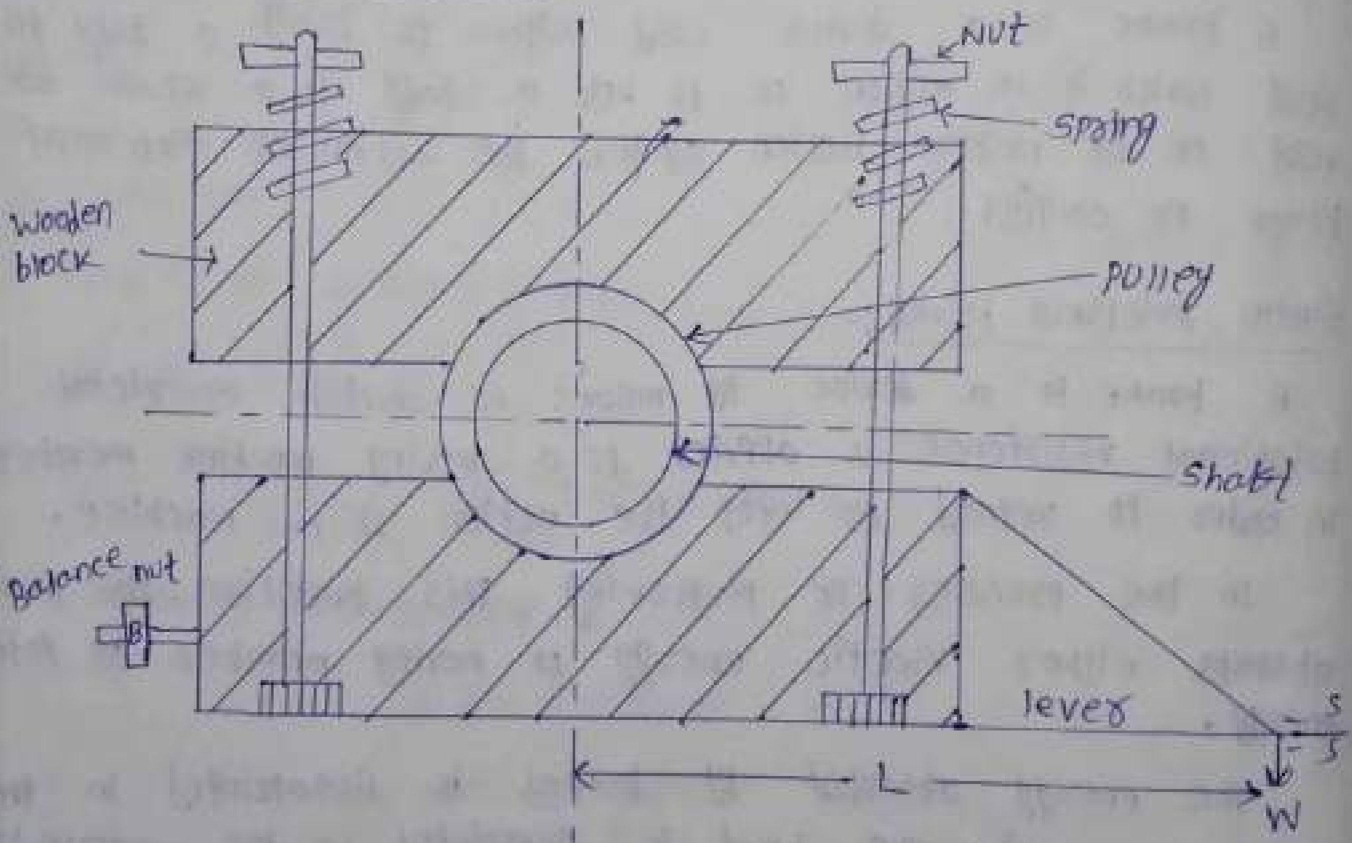
and driven member moving together, where as brakes are used to stop a moving member or to control its speed.

Absorption dynamometer

Absorption dynamometers consist of some form of brakes in which provision is made for measuring the frictional torque on the drum. These are mainly two types of absorption dynamometer :-

(i) prony brake dynamometer
(ii) Rope brake dynamometer

(i) prony brake dynamometer :-



It consists of two wooden blocks placed around a pulley fixed to shaft of an engine whose power is required to be measured. Each of wooden block embraces rather than one of the pulley rim. The two blocks can be drawn together by means of bolts, nuts and springs.

The lower block carries an arm/lever to the end of which a weight 'W' can be applied. A second arm projects from the block in the opposite division and carries a balance weight B, which balance the brake when loaded. Two stops are provided and the lever arm will float between these stops.

Let, W = weight at the end of lever.

R = Radius of pulley.

μ = coefficient of friction between pulley and block.

N = speed of shaft in rpm.

L = Horizontal distance of weight 'W' from the center of pulley.

Torque on the shaft (T) = W × L

Power of the engine (P) = T × ω

$$= T \times \frac{2\pi N}{60}$$

$$\boxed{\Rightarrow P = W \times L \times \frac{2\pi N}{60}} \text{ watt}$$

(ii) Rope brake dynamometer:-

It consists of one, two or more ropes wound round the rim of a pulley fixed rigidly to the shaft of the engine whose power is required to be measured. The upper end of the rope is attached to spring balance (S) while the lower end carries dead weight 'W'. The ropes are spaced evenly across the width of the rim by means of three or four wooden blocks at different point round the rim.

For measuring power of an engine the engine is made to run at constant speed under this condition, the torque transmitted by the engine must be equal to the frictional torque due to ropes.

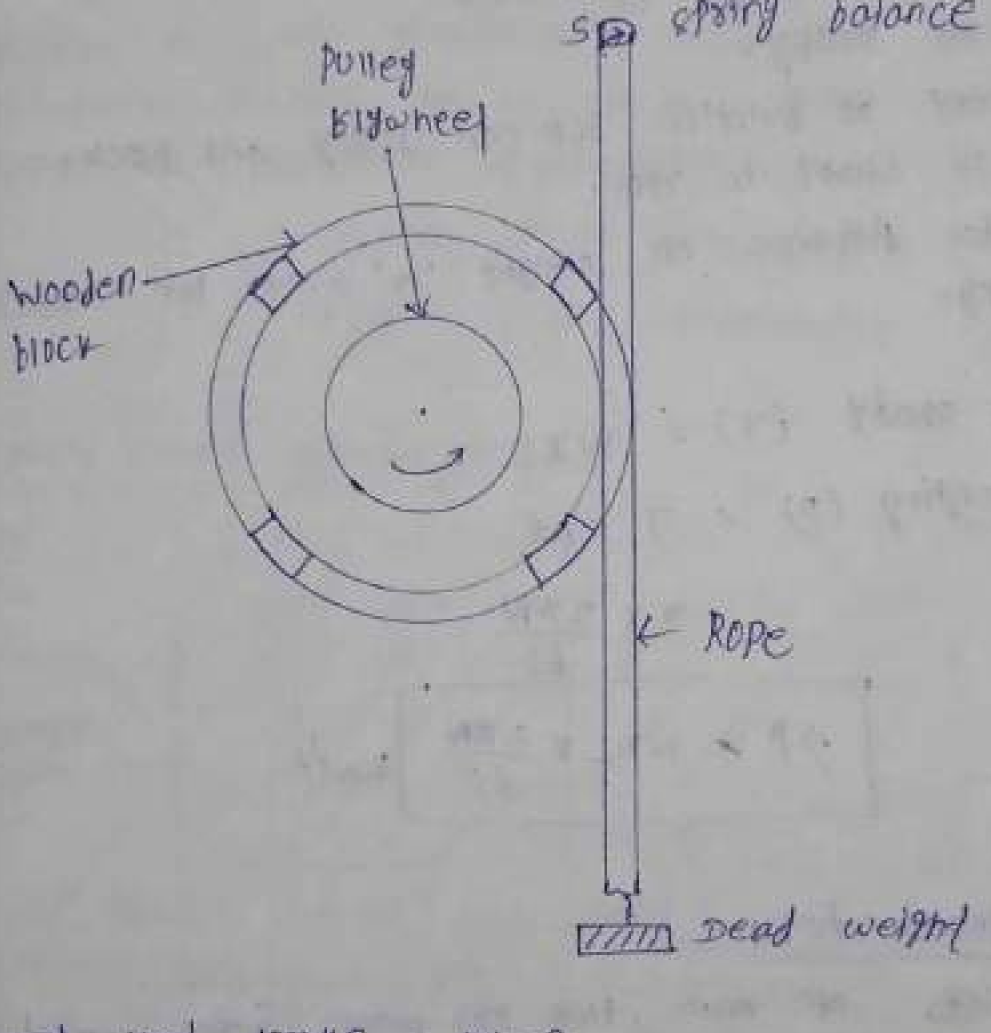
Let, N = constant speed of engine shaft.

W = Dead weight.

S = Spring balance reading.

D = Diameter of pulley rim.

d = diameter of rope



Net load brake = W - S

Frictional torque = Net load x distance of load line from center of shaft

$$= (W-S) \times \left(\frac{D+d}{2}\right)$$

Power = Torque x ω

$$= (W-S)\left(\frac{D+d}{2}\right) \times \omega$$

$$P = (W-S)\left(\frac{D+d}{2}\right)\frac{2\pi N}{60}$$

If diameter of rope is neglected,

$$P = (W-S)\frac{D}{2} \times \frac{2\pi N}{60}$$

$$\boxed{P = (W-S) \times R \times \frac{2\pi N}{60}}$$

Dynamometer

A dynamometer is a brake but in addition it has a device to measure the frictional resistance.

Dynamometers

- Absorption dynamometer
  - Prony brake dynamometer
  - Rope brake dynamometer
- Transmission dynamometer

→ In absorption dynamometers, the entire energy or power ~~power~~ produced by the engine is absorbed by the friction resistances of the brake and transformed in to heat, during the process of measurement.

→ In the transmission dynamometers, the energy is not wasted in friction but is used for doing work.

06.04.23

UNIT-3

The belts, ropes are used to transmit power from one shaft to another. By means of pulleys which rotates at same speed or at different speed. The belts and ropes are running over the pulleys. The pulleys are mounted on the two shafts.

Belts, ropes and chains are used where the distance between the two shafts is large. For small distance gears are used.

* Selection of belt drive :-

1. center distance between the shafts.
2. power to be transmitted.
3. speed of driving and driven shaft.
4. speed reduction ratio.

* Types of belt drive :-

1. Light belt drives :-

These are used to transmit small power at belt speed up to 10 m/s.

Ex- agricultural machine, small machine tool.

2. Medium drives :-

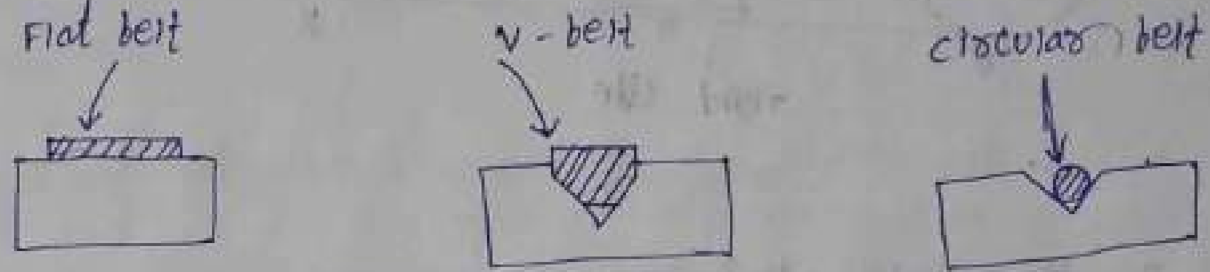
These are used to transmit medium power at belt speed over 10 m/s, but up to 22 m/s.

3. Heavy drives :-

These are used to transmit large or heavy power at belt speed above 22 m/s.

Ex :- used in compressor and generator.

(*) Types of Belts



① Flat belt :-

It is mostly used in factories and workshop where a moderate amount of power is to be transmitted from one pulley to another. when the two pulley are not more than 8m. apart.

② V - belt :-

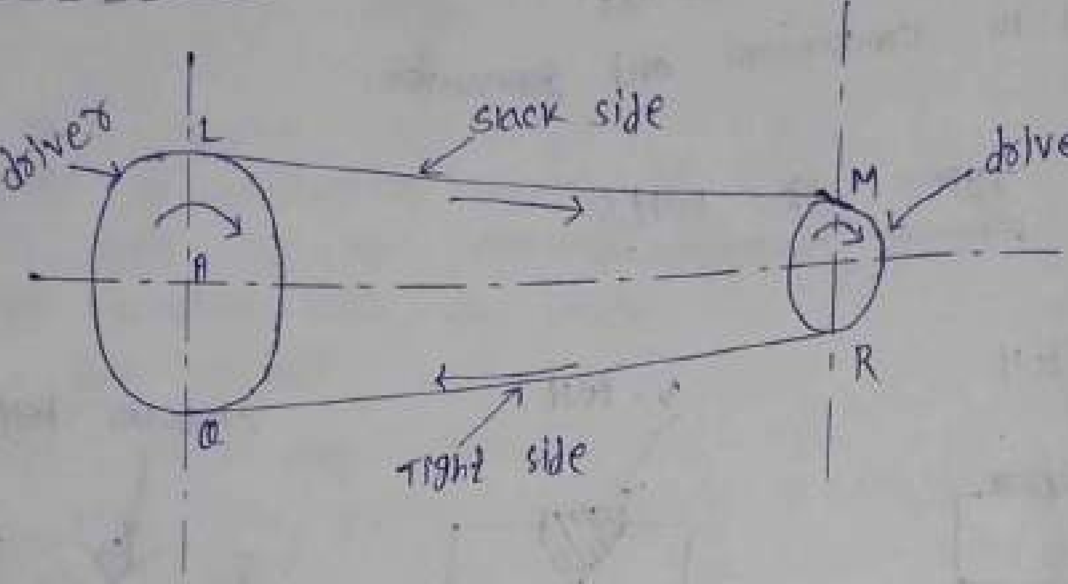
It is mostly used in workshop and factories where a moderate amount of power is to be transmitted from one pulley to another, when the two pulley are very nearer to each other.

③ circular belt or rope :-

It is mostly used in workshops and factories where a great amount of power is to be transmitted from one pulley to another, when the two pulley are more than 8m. apart.

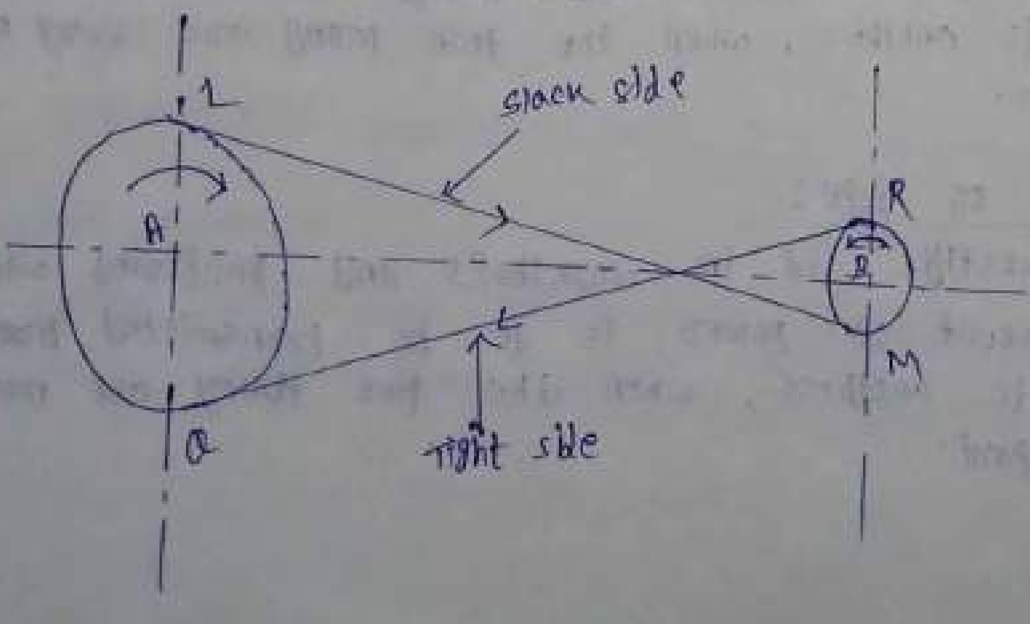
* Types of belt drives

1. Open belt drives :-



It is used with shafts arranged parallel and rotating in same direction. In this case, the driver A pulls the belt from one side (i.e lower side RQ) and delivers it to the other side (i.e upper side LM). Thus the tension in the lower side of the belt will be more than that in the upper side. The lower side of belt due to more tension is known as tight side, where upper side belt due to less tension is known as slack side.

2. Crossed / twist belt drive :-



The crossed or twist belt drive is used when shafts are arranged parallel and rotating in opposite direction. In this case the driver pulls the belt from one side (i.e RQ and delivers it to other side i.e LM. Thus tension in the belt RQ will be more than that in the belt LM.

→ The belt RQ due to more tension is known as tight side where as the belt LM due to less tension is known as slack side.

⊛ Velocity ratio of open belt drive

The ratio of the velocity of follower or driven to the velocity of driver, is known as velocity ratio.

Let, N_1 = speed of the driver in rpm

N_2 = speed of the driven in rpm

d_1 = diameter of driver in mm

d_2 = diameter of driven in mm

Let us consider length of belt that passes over the driver and follower in one minute.

Length of belt passing over the driver in one minute

= circumference of driver × No of revolution per minute

$= \pi d_1 \times N_1$

Length of belt passing over the driven or follower in one minute

= circumference of driven × No of revolution per minute

$= \pi d_2 \times N_2$

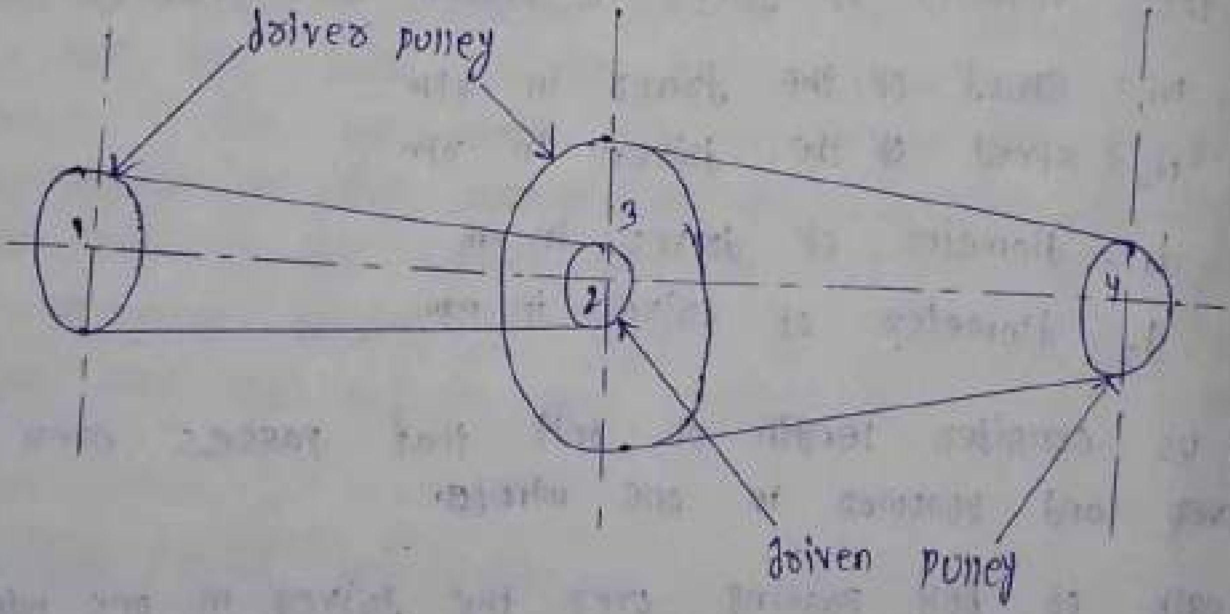
But length of belt passing over driver in one minute is equal to the length of belt passing over follower or driven in one minute.

$$\Rightarrow \pi d_1 N_1 = \pi d_2 N_2$$

$$\Rightarrow \frac{N_2}{N_1} = \frac{d_1}{d_2} = \text{velocity ratio.}$$

(*) Compound belt drive

When ~~the~~ power is to be transmitted from one shaft to another through a no. of pulley then a compound belt drive is used.



Let, N_1 = speed of pulley 1 in rpm

d_1 = diameter of pulley 1

N_2, d_2 = speed and diameter of pulley 2

N_3, d_3 = speed and diameter of pulley 3

N_4, d_4 = speed and diameter of pulley 4 velocity ratio of pulley 1 and 2 given by

$$\Rightarrow \frac{N_2}{N_1} = \frac{d_1}{d_2} \quad --- (i)$$

similarly velocity ratio of pulley 3 and 4 is given by

$$\Rightarrow \frac{N_4}{N_3} = \frac{d_3}{d_4} \quad --- (ii)$$

Multiplying eqn. (i) and (ii) is given by

$$\Rightarrow \frac{\cancel{N_2}}{N_1} \times \frac{N_4}{\cancel{N_3}} = \frac{d_1}{d_2} \times \frac{d_3}{d_4} \quad (\because N_2 = N_3)$$ as they mount on same shaft

$$\Rightarrow \frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4}$$

$$\frac{\text{speed of last driven}}{\text{speed of first driver}} = \frac{\text{product of diameter of driver}}{\text{product of diameter of driven}}$$

11-08-2023

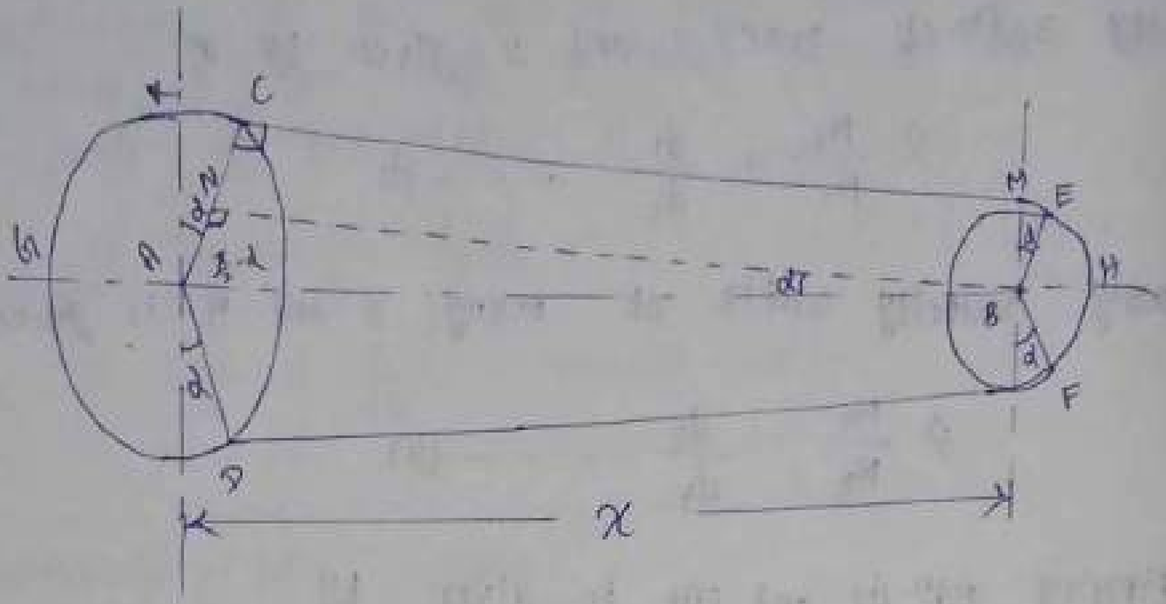
⊛ Length of belt :-

The length of belt means total length of belt required to connect a driver and follower.

There are two case to be considered :-

(i) Length of an open belt drive :-

The total length of an open belt equal to the length of belt not in contact with either pulley + the length of belt in contact with larger pulley + the length of belt contact with smaller pulley.



Let,

x = Distance between centres of two pulleys (length of AB).

r_1 = Radius of larger pulley

r_2 = Radius of smaller pulley

The belt leaves the larger pulley at C and D, and smaller pulley at E and F. Join C and D with A and Join E and F with B.

From B draw BN parallel to EC, but CE is tangent at C. Hence AC is perpendicular to CE, which means $\angle ACE = 90°$. As BN is parallel to CE hence $\angle ANB = 90°$

Let, $\angle ABN = \alpha$

$\angle BAN = 90° - \alpha = \frac{\pi}{2} - \alpha$

but $\angle BAK = 90° = \frac{\pi}{2}$

hence $\angle KAC = \angle BAK - \angle CAB$

$= \frac{\pi}{2} - \left(\frac{\pi}{2} - \alpha\right)$

$= \cancel{\frac{\pi}{2}} - \cancel{\frac{\pi}{2}} + \alpha = \alpha = \angle MBE$

Now $AN = AC - ACN$

$= AC - BE$

$= r_1 - r_2$

$BN = \sqrt{AB^2 - AN^2}$

$= \sqrt{x^2 - (r_1 - r_2)^2} = CE$

From ΔABN, $\sin\alpha = \frac{AN}{AB}$

$= \frac{r_1 - r_2}{x}$

here α is very small, so $\sin\alpha = \alpha$.

$\therefore \alpha = \frac{r_1 - r_2}{x}$

Total length of belt is given by,

$L = \text{Arc } DGC + CE + \text{Arc } EHF + FD$

$= 2 \times \text{Arc } GC + CE + FD + 2 \times \text{Arc } EH$

$= 2(\text{Arc } GC + CE + \text{Arc } EH) \quad (\because CE = FD)$

$= 2\left[r_1\left(\frac{\pi}{2} + \alpha\right) + \sqrt{x^2 - (r_1 - r_2)^2} + r_2\left(\frac{\pi}{2} - \alpha\right)\right] \cdots \text{(i)}$

$= 2\left[r_1\left(\frac{\pi}{2} + \alpha\right) + r_2\left(\frac{\pi}{2} - \alpha\right) + \sqrt{x^2 - (r_1 - r_2)^2}\right]$

$= 2\left[r_1 \times \frac{\pi}{2} + r_1\alpha + r_2 \times \frac{\pi}{2} - r_2\alpha + \sqrt{x^2 - (r_1 - r_2)^2}\right]$

$= 2\left[\frac{\pi}{2} r_1 + \frac{\pi}{2} r_2 + \alpha r_1 - \alpha r_2 + \sqrt{x^2 - (r_1 - r_2)^2}\right]$

$= 2\left[\frac{\pi}{2}(r_1 + r_2) + \alpha(r_1 - r_2) + \sqrt{x^2 - (r_1 - r_2)^2}\right] \cdots \text{(i}$

As $\sqrt{x^2 - (r_1-r_2)^2}$

$$= \sqrt{x^2 \left[1 - \frac{(r_1-r_2)^2}{x^2}\right]}$$

$$= \left[x^2\left\{1 - \left(\frac{(r_1-r_2)}{x}\right)^2\right\}\right]^{1/2}$$

By using binomial expansion we get.

$$= x\left[1 - \frac{1}{2}\left(\frac{r_1-r_2}{x}\right)^2 + \text{------}\right]$$

Neglecting smaller term

$$= x\left[1 - \frac{(r_1-r_2)^2}{2x^2}\right]$$

Substituting the value in eqn (i)

$$= 2\left[\frac{\pi}{2}(r_1+r_2) + \alpha(r_1-r_2) + x\left[1 - \frac{(r_1-r_2)^2}{2x^2}\right]\right]$$

$$= 2\left[\frac{\pi}{2}(r_1+r_2) + \alpha(r_1-r_2) + x - \frac{(r_1-r_2)^2}{2x}\right]$$

$$= \pi(r_1+r_2) + 2\alpha(r_1-r_2) + 2x - \frac{(r_1-r_2)^2}{x}$$

but we have, $\alpha = \frac{r_1-r_2}{x}$

putting the value of α we get.

$$= \pi(r_1+r_2) + 2\times\left(\frac{r_1-r_2}{x}\right)\times(r_1-r_2) + 2x - \frac{(r_1-r_2)^2}{x}$$

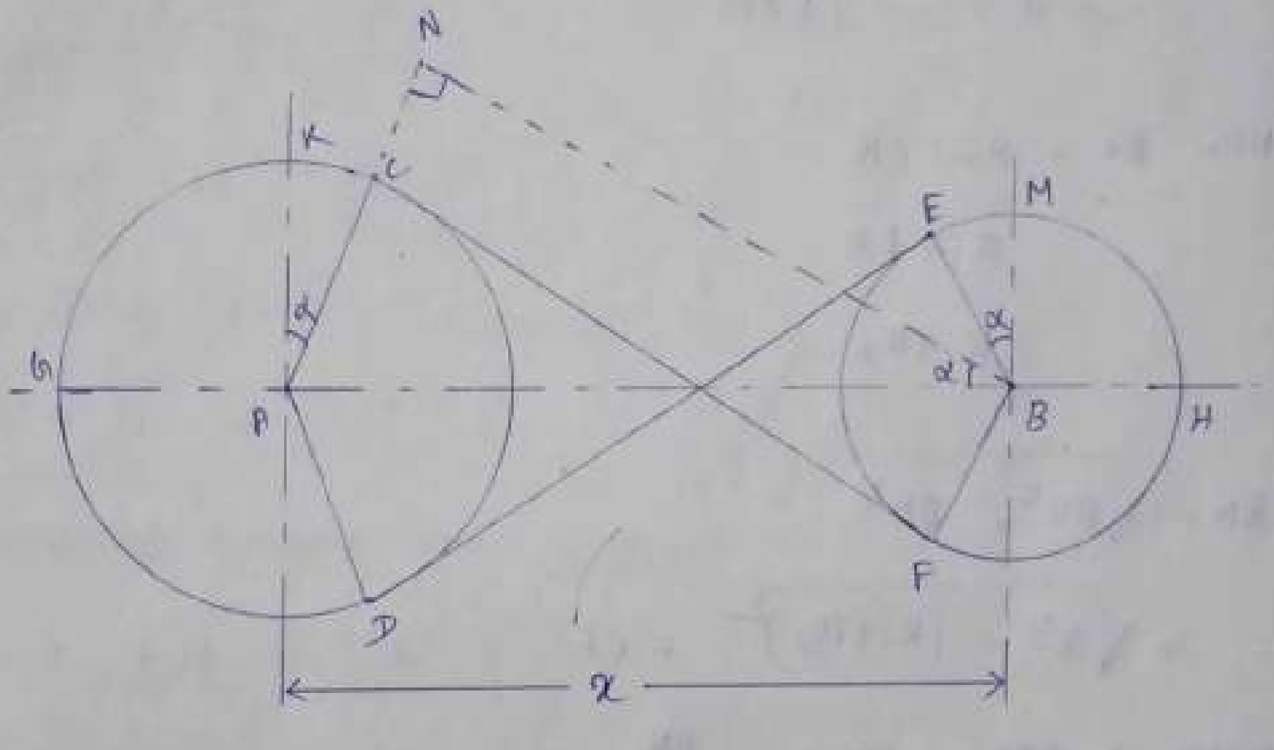
$$= \pi(r_1+r_2) + 2\times\frac{(r_1-r_2)^2}{x} + 2x - \frac{(r_1-r_2)^2}{x}$$

$$\boxed{L = \pi(r_1+r_2) + \frac{(r_1-r_2)^2}{x} + 2x.}$$

This is the approximet length of open belt drive, the length of belt depends of both the sum and difference of radii.

12.04.23

(ii) Length of cross belt drive



Let,

x = distance between the centers of two pulley or length of AB.

r_1 = Radius of larger pulley

r_2 = Radius of smaller pulley

L = Total length of cross belt

From B draw BN parallel to FC. But FC is tangent at C. Hence AC is perpendicular FC, which means

$\angle ACF = \angle ANB = 90^\circ$.

Let $\angle ABN = \alpha$

then $\angle BAN = 90^\circ - \alpha = \pi/2 - \alpha$

but $\angle BAK = 90^\circ = \pi/2$

$\angle KAC = \frac{\pi}{2} - \left(\frac{\pi}{2} - \alpha\right)$

$= \cancel{\frac{\pi}{2}} - \cancel{\frac{\pi}{2}} + \alpha$

$= \alpha = \angle EBM$

Now $AN = AC + CN$

$= r_1 + FB$

$= r_1 + r_2$

$BN = \sqrt{AB^2 - AN^2}$

$= \sqrt{x^2 - (r_1 + r_2)^2} = CF$

From ΔABN, $\sin\alpha = \frac{AN}{AB}$

here α is very small so $\sin\alpha = \alpha$

$\Rightarrow \alpha = \frac{AN}{AB} = \frac{r_1 + r_2}{x}$

Total length of belt is given by

$L = \text{Arc } DGC + CF + \text{Arc } FHE + ED$

$= 2 \times \text{Arc } GC + 2CF + 2 \times \text{Arc } FH$

$= 2(\text{Arc } GC + CF + \text{Arc } FH)$

$$= 2\left[r_1\left(\frac{\pi}{2}+\alpha\right) + \sqrt{x^2-(r_1+r_2)^2} + r_2\left(\frac{\pi}{2}+\alpha\right) \right]$$

$$= 2\left[\frac{\pi}{2} r_1 + \alpha r_1 + \sqrt{x^2-(r_1+r_2)^2} + \frac{\pi}{2} r_2 + \alpha r_2 \right]$$

$$= 2\left[\frac{\pi}{2} r_1 + \frac{\pi}{2} r_2 + \alpha r_1 + \alpha r_2 + \sqrt{x^2-(r_1+r_2)^2} \right]$$

$$= 2\left[\frac{\pi}{2}(r_1+r_2) + \alpha(r_1+r_2) + \sqrt{x^2-(r_1+r_2)^2} \right] \quad \text{---- (i)}$$

As $\sqrt{x^2-(r_1-r_2)^2}$

$$= \sqrt{x^2\left[1-\frac{(r_1+r_2)^2}{x^2}\right]}$$

$$= \left[x^2\left[1-\left(\frac{r_1+r_2}{x}\right)^2\right]\right]^{1/2}$$

by using binomial expansion we get

$$= x\left[1-\frac{1}{2}\left(\frac{r_1+r_2}{x}\right)^2 + \cdots\cdots\right]$$

neglecting smaller value or term

$$= x\left[1-\frac{(r_1+r_2)^2}{2x}\right]$$

substituting the value in eqn (i)

$$= 2\left[\frac{\pi}{2}(r_1+r_2) + \alpha(r_1+r_2) + x\left[1-\frac{(r_1+r_2)^2}{2x^2}\right] \right]$$

$$= 2\left[\frac{\pi}{2}(r_1+r_2) + \alpha(r_1+r_2) + x - \frac{(r_1+r_2)^2}{2x} \right]$$

$$= \pi(r_1+r_2) + 2\alpha(r_1+r_2) + 2x - \frac{(r_1+r_2)^2}{x}$$

but we have $\alpha = \dfrac{r_1+r_2}{x}$

Putting the value of α, we get

$$= \pi(r_1+r_2) + 2\,\frac{r_1+r_2}{x} \times r_1+r_2 + 2x - \frac{(r_1+r_2)^2}{x}$$

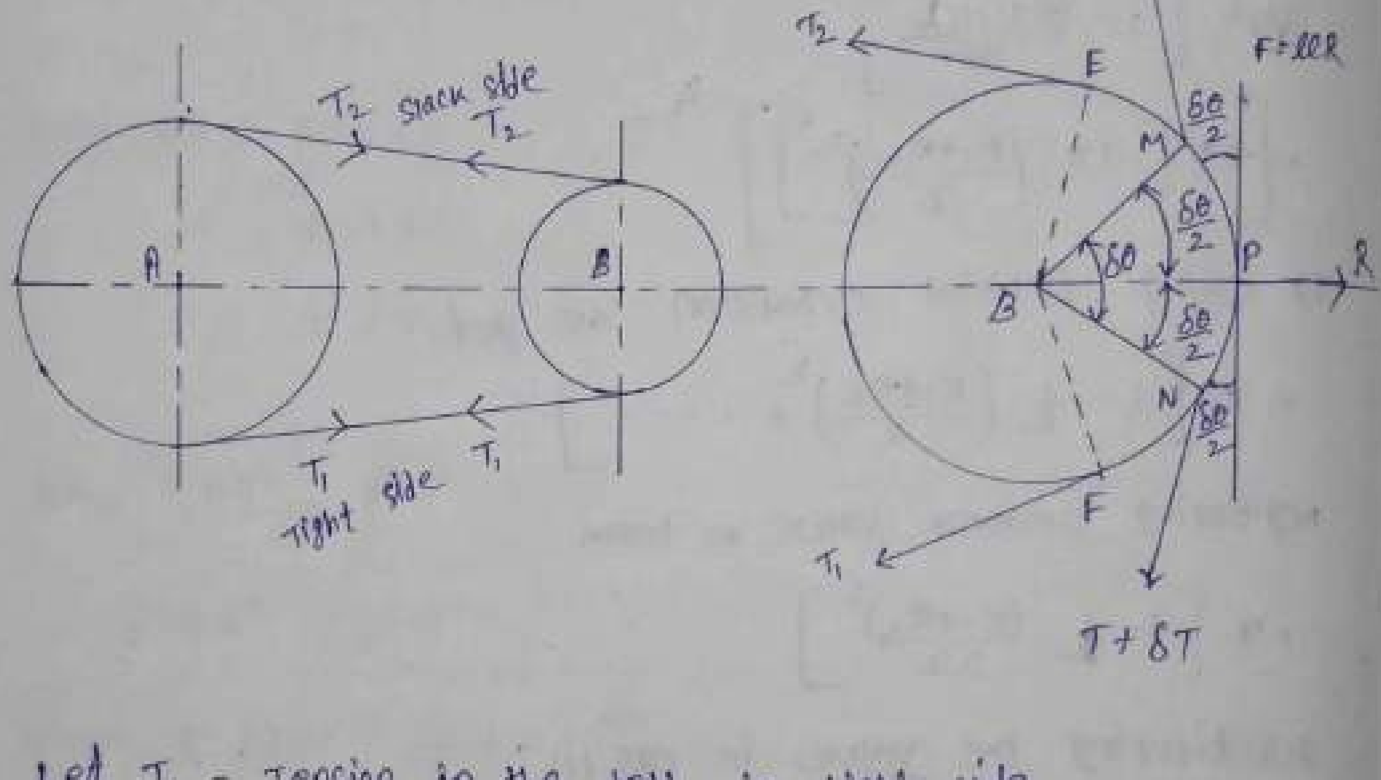
$$= \pi(r_1+r_2) + 2\,\frac{(r_1+r_2)^2}{x} + 2x - \frac{(r_1+r_2)^2}{x}$$

$$= \pi(r_1+r_2) + 2\frac{(r_1+r_2)^2}{x} - \frac{(r_1+r_2)^2}{x} + 2x$$

$$= \pi(r_1+r_2) + \frac{(r_1+r_2)^2}{x} + 2x.$$

19.04.23

(*) Ratio of belt tension



Let T_1 = Tension in the belt in tight side

T_2 = Tension in the belt in slack side

μ = coefficient of friction between belt and pulley

θ = angle of contact

The ratio of two tensions may be found by considering an elementary piece of the belt MN subtending an angle $\delta\theta$ at the centre of pulley B.

The various forces which keeps the elementary piece MN equilibrium are

(i) Tension T in the belt at M acting tangentially

(ii) Tension $(T + \delta T)$ in the belt at N acting tangentially

(iii) Normal reaction R acting radially outward at P. P is middle of MN.

(iv) Frictional force $F = \mu R$ acting at right angle to R and in opposite direction of motion of pulley.

$\angle PBM = \angle PBN = \frac{\delta\theta}{2}$

Resolving all forces acting on the belt in horizontal direction

$(T + \delta T)\sin\frac{\delta\theta}{2} + T\sin\frac{\delta\theta}{2} = R$ ------ (*)

since $\delta\theta$ is very small,

$\sin\frac{\delta\theta}{2} = \frac{\delta\theta}{2}$

$\Rightarrow R = (T + \delta T)\times\frac{\delta\theta}{2} + T\times\frac{\delta\theta}{2}$

$\Rightarrow R = T\frac{\delta\theta}{2} + \frac{\delta T\,\delta\theta}{2} + T\frac{\delta\theta}{2}$

Neglecting small quantity $\frac{\delta T \cdot \delta\theta}{2}$

$\Rightarrow R = 2T\frac{\delta\theta}{2}$

$\Rightarrow R = T\,\delta\theta$ ------- (i)

Resolving all forces acting on the belt in vertical direction

$T\cos\frac{\delta\theta}{2} + \mu R = (T + \delta T)\cos\frac{\delta\theta}{2}$

since $\delta\theta$ is very small, $\cos\frac{\delta\theta}{2}$ reduced to unity.

$\Rightarrow T + \mu R = T + \delta T$

$\Rightarrow \mu R = \delta T$

$\Rightarrow R = \frac{\delta T}{\mu}$ --- (ii)

From eqn (i) and (ii)

$\Rightarrow T\,\delta\theta = \frac{\delta T}{\mu}$

$\Rightarrow \frac{\delta T}{T} = \delta\theta\,\mu$

integrating the above eqn between limit T_2 to T_1

$\int_{T_2}^{T_1} \frac{\delta T}{T} = \int \delta\theta\,\mu$

$\Rightarrow \left[\ln\right]_{T_2}^{T_1} = \mu\theta$

$\Rightarrow \ln T_1 - \ln T_2 = \mu\theta$ $\left(\because \log a - \log b = \log\frac{a}{b}\right)$

$\Rightarrow \ln\frac{T_1}{T_2} = \mu\theta$

$\boxed{\Rightarrow \frac{T_1}{T_2} = e^{\mu\theta}}$.

Angle of contact for open belt drive

1. $\theta = 180 - 2\alpha$

2. $\sin\alpha = \frac{r_1 - r_2}{x}$

where, r_1 = Radius of large pulley
r_2 = Radius of small pulley
x = distance between centre of two pulley.

Angle of contact for cross belt drive

1. $\theta = 180 + 2\alpha$

2. $\sin\alpha = \frac{r_1 + r_2}{x}$

Power transmitted by belt

T_1 = Tension in the tight side belt

T_2 = Tension in the slack side belt

V = velocity of belt in m/sec

Effecting driving force(F) = $T_1 - T_2$

Workdone = $F \times V$

$$P = (T_1 - T_2) \times V \text{ (watt)}$$

$$P = \frac{(T_1 - T_2) V}{1000} \text{ kW.}$$

① A belt is running over a pulley of diameter 120 cm at 200 rpm. The angle of contact is 165° and coefficient of friction between the belt and pulley is 0.3. If the maximum tension in the belt is 3000 N, find the power transmitted by belt.

Ans: Given data,

$D = 120\text{ cm} = 1.2\text{m}$

$N = 200\text{ rpm}$

$\theta = 165°$ Radian: $\frac{\pi}{180} \times 165 = 2.87$

$\mu = 0.3$

$T_1 = 3000\text{ N}$

$P = ?\ \frac{(T_1 - T_2)V}{1000}$

we know, $\frac{T_1}{T_2} = e^{\mu\theta}$

$\Rightarrow T_2 = \frac{T_1}{e^{\mu\theta}} = \frac{3000}{e^{0.3 \times 2.87}} = 1268.21\text{N}$

$V = \frac{\pi D N}{60} = \frac{\pi \times 1.2 \times 200}{60} = 12.56$

$\text{Power}(P) = \frac{(T_1 - T_2)V}{1000}$

$= \frac{(3000 - 1268.21) \times 12.56}{1000}$

$= 21.75\text{ kW. (Ans)}$

(2) An open belt drive connects two pulleys 120 cm and 50 cm diameters, on parallel shafts 4m. apart. The maximum tension in the belt is 1855.3 N. The coefficient of friction is 0.3. The driver pulley of diameter 120 cm, runs at 200 rpm. calculate
(i) power transmitted
(ii) torque on each of two shafts.

Ans: Given data,

$d_1 = 120 \text{ cm} = 1.2\text{m}$, $r_1 = 0.6 \text{ m}$

$d_2 = 50 \text{ cm} = 0.5\text{m}$, $r_2 = 0.25\text{m}$

$T_1 = 1855.3 \text{ N}$

$\mu = 0.3$

$N = 200 \text{ rpm}$

$x = 4\text{m}.$

(i) power transmitted

$$P = \frac{(T_1 - T_2) V}{1000}$$

we know for open belt drive

$$\sin\alpha = \frac{r_1 - r_2}{x}$$

$$= \frac{0.6 - 0.25}{4} = 0.0875$$

$$\Rightarrow \alpha = \sin^{-1} 0.0875 = 5.019^\circ$$

$$\therefore \theta = 180 - 2\alpha$$

$$= 180 - (2 \times 5.019) = 169.96^\circ$$

$$\text{Radian} = \frac{\pi}{180} \times 169.96 = 2.96 \text{ radian}.$$

$$\therefore T_2 = \frac{T_1}{e^{\mu\theta}}$$

$$= \frac{1855.3}{e^{0.3 \times 2.96}} = 763.41 \text{ N}.$$

$$V = \frac{\pi d N}{60}$$

$$= \frac{\pi \times 1.2 \times 200}{60} = 12.56 \text{ m/s}$$

$$\therefore P = \frac{(T_1 - T_2) V}{1000} = \frac{(1855.3 - 763.41) \times 12.56}{1000} = 13.71 \text{ kW.}$$

(ii) Torque on each two shafts

Torque on the driver pulley

$$T_1 = (T_1 - T_2) \times R_1 \quad \text{or} \quad F \times R_1$$

$$= (1855.3 - 763.41) \times 0.6$$

$$= 655.13.$$

Torque on the driven pulley

$$F = (T_1 - T_2)$$

$$= (1855.3 - 763.41)$$

$$= 1091.89.$$

$$T_2 = F \times R_2$$

$$= 1091.89 \times 0.25$$

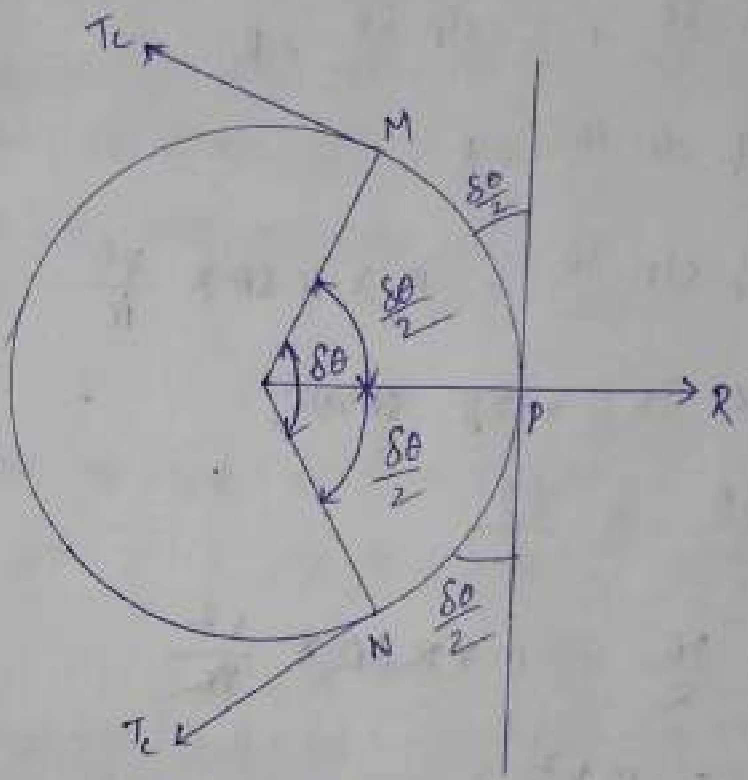
$$= 272.97.$$

20.04.23

* Centrifugal tension

The tension caused in the running belt by the centrifugal force is known as centrifugal tension.

→ Whenever a particle or mass M is rotated in a circular path of radius 'r' at a uniform velocity 'v', a centrifugal force is acting radially outward and its magnitude is $\frac{mv^2}{r}$.



Let, v = velocity of belt in m/s

r = radius of pulley over which belt runs

m = Mass of belt per meter length

T_c = centrifugal tension acting at 'M' and 'N' tangentially

R = centrifugal force acting radially outward

M = Mass of elementary length of belt MN

Centrifugal force acting radially outward is balanced by components of T_c acting radially inward.

$MN = r \times \delta\theta$

Mass of belt MN = Mass per meter length x length of MN

$\Rightarrow M = m \times r\delta\theta$

centrifugal force $\Rightarrow R = \frac{Mv^2}{r}$

$= m \times r\delta\theta \times \frac{v^2}{r}$

Resolving forces on horizontally

$T_c \sin\frac{\delta\theta}{2} + T_c \sin\frac{\delta\theta}{2} = R$

$\Rightarrow 2T_c \sin\frac{\delta\theta}{2} = R$.

$\Rightarrow 2T_c \sin\frac{\delta\theta}{2} = m \times r\delta\theta \times \frac{v^2}{r}$

as $\delta\theta$ is very small;

$\sin\frac{\delta\theta}{2} = \frac{\delta\theta}{2}$

$\Rightarrow \cancel{2} T_c \frac{\cancel{\delta\theta}}{\cancel{2}} = m \times \cancel{r\delta\theta} \times \frac{v^2}{\cancel{r}}$

$\Rightarrow T_c = mv^2$.

* If centrifugal tension is consider

For tight side $= T_1 + T_c$

For slack side $= T_2 + T_c$

* Maximum tension $(T_m) = f \times (b \times t)$

= Maximum shape stress x cross sectional area

$T_m = T_1 + T_c$, if the centrifugal tension is considered.

$T_m = T_1$, if centrifugal tension is neglected.

(*) Initial tension in the belt

Tension in the belt which is passing over the two pulleys (i.e driver and driven), when the pulleys are stationary is known as initial tension in the belt.

When power is supplied to one of the pulley and transmitted to other, the tension in the two free length of belt will be changed. The tight side of the belt stretches until the pull is increased from T_0 to T_1 and slack side shortens until the pull is decreased from T_0 to T_2.

Increase of tension in tight side $= T_1 - T_0$

decrease of tension in slack side $= T_0 - T_2$

increase in length of belt in tight side =

$$= \alpha \times \text{increase in tension}$$

$$= \alpha \times (T_1 - T_0)$$

decrease in length of belt in slack side

$$= \alpha \times \text{decrease in tension}$$

$$= \alpha \times (T_0 - T_2)$$

For perfectly elastic material of the belt, length of belt remain constant when it is rest or at motion.

increase in length of belt in tight side = decrease length of belt in slack side

$$\Rightarrow \alpha(T_1 - T_0) = \alpha(T_0 - T_2)$$

$$\Rightarrow T_1 - T_0 = T_0 - T_2$$

$$\Rightarrow T_1 + T_2 = 2T_0$$

$$\Rightarrow T_0 = \frac{T_1 + T_2}{2}$$

$$\boxed{\Rightarrow T_0 = \frac{T_1 + T_2 + 2T_c}{2}}$$ if centrifugal tension is consider.

H.W

① An open belt drive running over two pulleys 1.5m and 1.0m diameters connects two parallel shafts 4.80m apart. The initial tension in the belt when stationary is 3000N. If the smaller pulley is rotating at 600 rpm and coefficient of friction between belt and pulley is 0.3, determine the power transmission taking centrifugal tension in to account. The mass of belt is given as 0.6703 kg/m length.

Ans: Given data,

$D_1 = 1.5m$

$D_2 = 1.0m$

$x = 4.80m$

$T_0 = 3000N$

$N_2 = 600 rpm$

$\mu = 0.3$

$m = 0.6703$ kg/m

$P = ? \frac{(T_1 - T_2) v}{1000}$

$T_c = ? mv^2$

$\therefore v = \frac{\pi D_2 N_2}{60} = \frac{\pi \times 1.0 \times 600}{60} = 31.41$ m/s

$T_c = mv^2 = 0.6703 \times (31.41)^2 = 661.31$ N.

We know, $T_0 = \frac{T_1 + T_2 + 2T_c}{2}$

$\Rightarrow 3000 = \frac{T_1 + T_2 + (2 \times 661.31)}{2}$

$\Rightarrow T_1 + T_2 = 3000 \times 2 - (2 \times 661.31)$

$\Rightarrow T_1 + T_2 = 4677.38$ ------ (i) we know, $\sin\alpha = \frac{r_1 - r_2}{x} = \frac{0.75 - 0.5}{4.80} = 0.052$

$\alpha = \sin^{-1} 0.052 = 2.980$

$\theta = 180 - 2\alpha = 180 - (2 \times 2.980) = 174.04$

radian $= \frac{\pi}{180} \times 174.04 = 3.03$

$\theta = 3.03$ rad

we know $\frac{T_1}{T_2} = e^{\mu\theta}$

$\Rightarrow T_1 = T_2 \times e^{0.3 \times 3.03}$

$\Rightarrow T_1 = T_2\, 2.481$ ---- (ii)

putting the value in eqn (i) we get

$\Rightarrow T_1 + T_2 = 4677.38$

$\Rightarrow T_2 \times 2.481 + T_2 = 4677.38$

$\Rightarrow T_2 (2.481 + 1) = 4677.38$

$\Rightarrow T_2 = \frac{4677.38}{3.481} = 1343.68$

$\therefore T_1 + T_2 = 4677.38$

$\Rightarrow T_1 = 4677.38 - 1343.68$

$\Rightarrow T_1 = 3333.7$

$\therefore$ power (P) $= \frac{(T_1 - T_2) V}{1000}$

$= \frac{(3333.7 - 1343.68) \times 31.41}{1000}$

$= 62.54$ kW. (Ans)

21.04.23

Effect of belt thickness on velocity ratio

If the belt thickness is consider, then the mean diameter of rotation will be equal to the diameter of driver or follower plus thickness of belt.

Since mean radius of rotation will became as the radius of driver or follower plus half belt thickness.

If there is no sleep between belt and pulley, then the perimeter speed of two pulley should be same. This should be equal to the velocity of belt.

$$\boxed{V = \omega_1 r_{m_1} = \omega_2 r_{m_2}} \quad \text{------(i)}$$

where, ω_1 = angular velocity of driver $= \frac{2\pi N_1}{60}$

ω_2 = angular velocity of driven $= \frac{2\pi N_2}{60}$

r_{m_1} = Mean radius of rotation of driver pulley

$= r_1 + \frac{t}{2}$

r_{m_2} = Mean radius of rotation of driven pulley

$= r_2 + \frac{t}{2}$

t = thickness of belt.

From eqn(i) $\Rightarrow \omega_1 \times \left(r_1 + \frac{t}{2}\right) = \omega_2 \times \left(r_2 + \frac{t}{2}\right)$

$$\Rightarrow \frac{\omega_2}{\omega_1} = \frac{r_1 + \frac{t}{2}}{r_2 + \frac{t}{2}}$$

$$\Rightarrow \frac{\frac{\cancel{2\pi} N_2}{\cancel{60}}}{\frac{\cancel{2\pi} N_1}{\cancel{60}}} = \frac{\frac{2r_1 + t}{\cancel{2}}}{\frac{2r_2 + t}{\cancel{2}}}$$

$$\Rightarrow \frac{N_2}{N_1} = \frac{2R_1+t}{2R_2+t}$$

$$\Rightarrow \frac{N_2}{N_1} = \frac{d_1+t}{d_2+t}$$

1. Find the speed of a shaft which is driven with the help of belt by an engine running at 200 rpm. The diameter of engine pulley 51 cm and that of shaft 30 cm.

Ans: Given data,

$N_1 = 200$ rpm

$d_1 = 51$ cm $= 0.51$ m

$d_2 = 30$ cm $= 0.30$ m

$N_2 = ?$

We know, $\frac{N_2}{N_1} = \frac{d_1}{d_2}$

$$\Rightarrow N_2 = \frac{d_1 \times N_1}{d_2}$$

$$\Rightarrow N_2 = \frac{0.51 \times 200}{0.30} = 340 \text{ rpm}.$$

2. If in the above problem the thickness of belt 10mm find the speed of the shaft.

Ans: Given data,

$N_1 = 200$ rpm

$d_1 = 51$ cm $= 0.51$ m

$d_2 = 30$ cm $= 0.30$ m

$N_2 = ?$

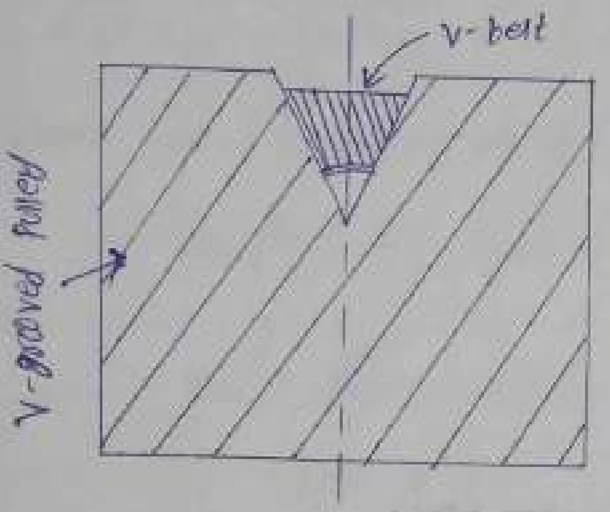
$t = 10$ mm $= 0.01$ m

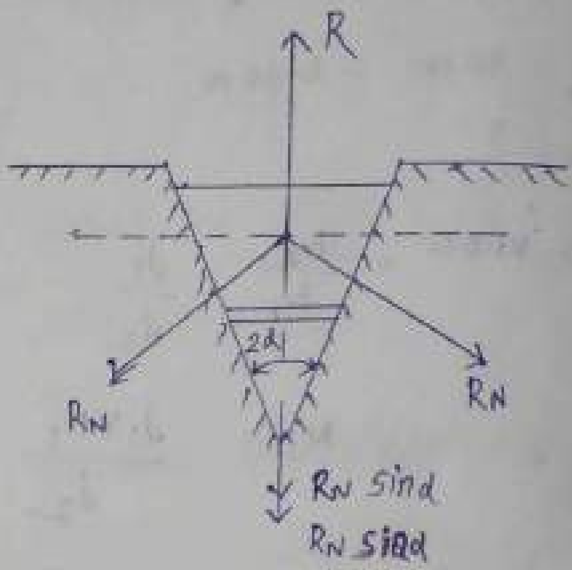
$$\therefore \frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}$$

$$\Rightarrow N_2 = \frac{N_1 \times (d_1 + t)}{d_2 + t}$$

$$= \frac{200 \times (0.51 + 0.01)}{0.3 + 0.01} = 335.48 \text{ rpm.}$$

V-Belt





Let, 2α = angle of groove

μ = co-efficient of friction

R_N = Normal reaction between belt and sides of V-grooved pulley.

R = Total reaction in the plain of groove,

Resolving forces in vertically

$R = R_N \sin\alpha + R_N \sin\alpha$

$\Rightarrow R = 2 R_N \sin\alpha$

$\Rightarrow R_N = \dfrac{R}{2\sin\alpha}$.

$\Rightarrow R_N = \cancel{\dfrac{R}{2\operatorname{cosec}\alpha}}\ \dfrac{R}{2} \times \operatorname{cosec}\alpha$

Frictional resistance $= \mu R_N + \mu R_N$

$= 2\mu R_N$

$= \cancel{2}\mu \times \dfrac{R}{\cancel{2}} \operatorname{cosec}\alpha$

$= \mu R \operatorname{cosec}\alpha$

For flat belt drive frictional force $= \mu R$

For 'V' belt drive frictional force $= (\mu \operatorname{cosec}\alpha) R$

23.04.23

③ Maximum power transmitted by a belt

Let,

T_1 = Tension on tight side

T_2 = Tension on slack side

v = linear velocity of belt

power transmitted (P) = $(T_1 - T_2)\, v$ ..... (i)

we know that, $\dfrac{T_1}{T_2} = e^{\mu\theta}$

$\Rightarrow T_2 = \dfrac{T_1}{e^{\mu\theta}}$

putting the value in eqn (i), we get

$\Rightarrow P = \left(T_1 - \dfrac{T_1}{e^{\mu\theta}}\right) v$

$= T\left(1 - \dfrac{1}{e^{\mu\theta}}\right) v$ ..... (ii)

Let, $1 - \frac{1}{e^{\mu\theta}} = K$

so eqn (ii) became

$P = T_1 \left(1 - \frac{1}{e^{\mu\theta}}\right) V$

$\Rightarrow P = T_1 K V$

$\Rightarrow P = K T_1 V$, ---- eqn (iii)

$T_{max} = T_1 + T_c$

$\Rightarrow T_1 = T_{max} - T_c$

putting the value of T_1 in eqn (iii)

$\Rightarrow P = K(T_{max} - T_c) V$

$= K(T_{max} - mV^2) V$

$= K(T_{max} V - mV^3)$

power transmitted will be maximum if $\frac{dP}{dV} = 0$

$\frac{dP}{dV} = 0$

$\Rightarrow \frac{d}{dV}\left[K(T_{max} V - mV^3)\right] = 0$

$\Rightarrow K\left[\frac{d}{dV} T_{max} V - \frac{d}{dV} mV^3\right] = 0$

$\Rightarrow T_{max} - 3mV^2 = 0$

$\Rightarrow T_{max} - 3T_c = 0$ $\quad (\because T_c = mV^2)$

$\Rightarrow T_{max} = 3T_c$.

again,

$T_{max} = T_1 + T_c$

$\Rightarrow T_{max} = T_1 + \frac{T_{max}}{3}$ $(\because T_{max} = 3T_c)$

$\Rightarrow T_1 = T_{max} - \frac{T_{max}}{3}$

$\Rightarrow T_1 = \frac{3T_{max} - T_{max}}{3}$

$\Rightarrow T_1 = \frac{2T_{max}}{3} = \frac{2}{3} T_{max}$

$\Rightarrow T_{max} = \frac{3}{2} T_1$.

NOTE

$\frac{T_1}{T_2} = e^{\mu\theta \operatorname{cosec} \alpha}$

(8) Slip of belt

When the driver pulley rotates, it carries the belt due to firm grip between its surface and belt. The firm grip between pulley and belt is obtained by friction. This firm grip is called friction grip. But sometimes the frictional grip is not sufficient. This may cause some forward motion of driver pulley without carrying belt with it. This means there is a relative motion between the driver pulley and belt. The difference between linear speed of the pulley rim and the belt is a measure of slip. Generally the slip is expressed as percentage

Let, N_1 = speed of driver, in rpm

N_2 = speed of follower, in rpm

s_1 = % slip between driver and belt

s_2 = % slip between follower and belt

v = velocity of belt, passing over driver pulley/min.

Perimeterial velocity of driver pulley

$\Rightarrow V_1 = \omega_1 \times r_1$

$$= \frac{2\pi N_1}{60} \times \frac{d_1}{2}$$

$$= \frac{\pi d_1 N_1}{60}.$$

due to slip between the driver pulley and the belt, the velocity of belt decrease.

The velocity of belt $= \frac{\pi d_1 N_1}{60} - \frac{\pi d_1 N_1}{60} \times \frac{s_1}{100}$

$$= \frac{\pi d_1 N_1}{60}\left(1 - \frac{s_1}{100}\right) \text{ ------ (i)}$$

Now this belt is passing over the follower and as there is a slip between the driven pulley and the belt, the velocity of follower pulley will be decrease so peripherial velocity of the follower

$$= \text{velocity of the belt} - \text{velocity of belt} \times \frac{s_2}{100}$$

$$= \frac{\pi d_1 N_1}{60}\left(1 - \frac{s_1}{100}\right) - \frac{\pi d_1 N_1}{60}\left(1 - \frac{s_1}{100}\right) \times \frac{s_2}{100}$$

$$= \frac{\pi d_1 N_1}{60}\left(1 - \frac{s_1}{100}\right) \cdot \left(1 - \frac{s_2}{100}\right) \text{ ------ (ii)}$$

But actually the perimeterial velocity of the follower

$\Rightarrow V_2 = \omega_2 \times r_2$

$$= \frac{2\pi N_2}{60} \times \frac{d_2}{2}$$

$$= \frac{\pi d_2 N_2}{60} \text{ ------ (iii)}$$

comparing eqⁿ (ii) and eqⁿ (iii), we get

$$\Rightarrow \frac{\pi d_1 N_1}{60}\left(1-\frac{s_1}{100}\right)\left(1-\frac{s_2}{100}\right) = \frac{\pi d_2 N_2}{60}$$

$$\Rightarrow d_1 N_1\left(1-\frac{s_1}{100}\right)\left(1-\frac{s_2}{100}\right) = d_2 N_2$$

$$\Rightarrow d_2 N_2 = d_1 N_1\left[1-\frac{s_2}{100}-\frac{s_1}{100}+\frac{s_1 \times s_2}{100}\right]$$

$$\Rightarrow d_2 N_2 = d_1 N_1\left[1-\frac{s_1+s_2}{100}+\frac{s_1 s_2}{100}\right]$$

as $\frac{s_1 s_2}{100}$ is very small so neglected this

$$\Rightarrow d_2 N_2 = d_1 N_1\left[1-\frac{s_1+s_2}{100}\right]$$

$$\Rightarrow d_2 N_2 = d_1 N_1\left[1-\frac{s}{100}\right] \quad \left[\because s_1+s_2 = s \text{ total slip}\right]$$

$$\Rightarrow \frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1-\frac{s}{100}\right)$$

Here thickness is not consider.

$$\Rightarrow \frac{N_2}{N_1} = \frac{d_1+t}{d_2+t}\left(1-\frac{s}{100}\right)$$

Here thickness is consider.

① The maximum allowable tension in a V belt of groove angle 45° is 1500 N. The angle of lap is 170° and coefficient of friction between the belt and material of pulley is 0.27. If the belt is running at 2 m/s. Determine (i) Net driving tension
(ii) power transmitted by the pulley, neglect effect of centrifugal tension.

Ans: Given data,

$2\alpha = 45°$, $\alpha = 22.5°$

$\theta = 170°$

radian $= \frac{\pi}{180} \times 170 = 2.96$

$\mu = 0.27$

$T_1 = T_{max} = 1500$ N

We know, $\frac{T_1}{T_2} = e^{\mu\theta \operatorname{cosec}\alpha}$

$$\Rightarrow T_2 = \frac{T_1}{e^{\mu\theta \operatorname{cosec}\alpha}}$$

$$\Rightarrow T_2 = \frac{T_1}{e^{0.27 \times 2.96 \times \operatorname{cosec} 22.5}}$$

$$\Rightarrow T_2 = 185.82 \text{ N}.$$

(i) Net driving tension $= (T_1 - T_2)$

$= 1500 - 185.82$

$= 1314.18$ N. (Ans)

(ii) power transmitted (P) = $(T_1 - T_2) \times V$

$= 1314.18 \times 2$

$= 2628.36$ W

$= 2.62$ kW. (Ans)

26.04.23

(2) A belt embraces the shorter pulley by an angle of 165° and runs at a speed of 1700 m/min. Dimensions of belt are width = 20 cm and thickness 8mm. Its density is 1 gm/cm³. Determine the maximum power that can be transmitted at the above speed, if the maximum permi stress in the belt is not exceed 250 N/cm² and $\mu = 0.25$.

Ans: Given data,

$\theta = 165° = 2.87$ radian

$v = 1700$ m/min $= \frac{1700}{60} = 28.33$ m/s

$b = 20$ cm

$t = 8$mm $= 0.8$ cm

$\rho = 1$ gm/cm³

$P = ?$

$\sigma = 250$ N/cm²

Maximum tension $(T_m) = \sigma \times$ area of belt

$= \sigma \times b \times t$

$= 250 \times 20 \times 0.8$

$= 4000$ N.

$\because \rho = \frac{M}{V}$

$M = \rho \times V$

$= 1\ g/cm^3 \times b \times t \times l$

$= 1\ g/cm^3 \times 20\ cm \times 0.8\ cm \times 100\ cm$

$= 1600\ gm$

$= 1.6\ kg.$

centrifugal tension $(T_c) = MV^2$

$= 1.6 \times 28.33^2$

$= 1284.14\ N$

$T_{max} = T_1 + T_c$

$\Rightarrow T_1 = T_{max} - T_c$

$= 4000 - 1284.14 = 2715.86\ N$

we know, $\frac{T_1}{T_2} = e^{\mu\theta}$

$\Rightarrow T_2 = \frac{T_1}{e^{\mu\theta}} = \frac{2715.86}{e^{0.25 \times 2.87}} = 1325.25\ N$

power transmitted (P) $= \frac{(T_1 - T_2) \times V}{1000}$

$= \frac{(2715.86 - 1325.25) \times 28.33}{100}$

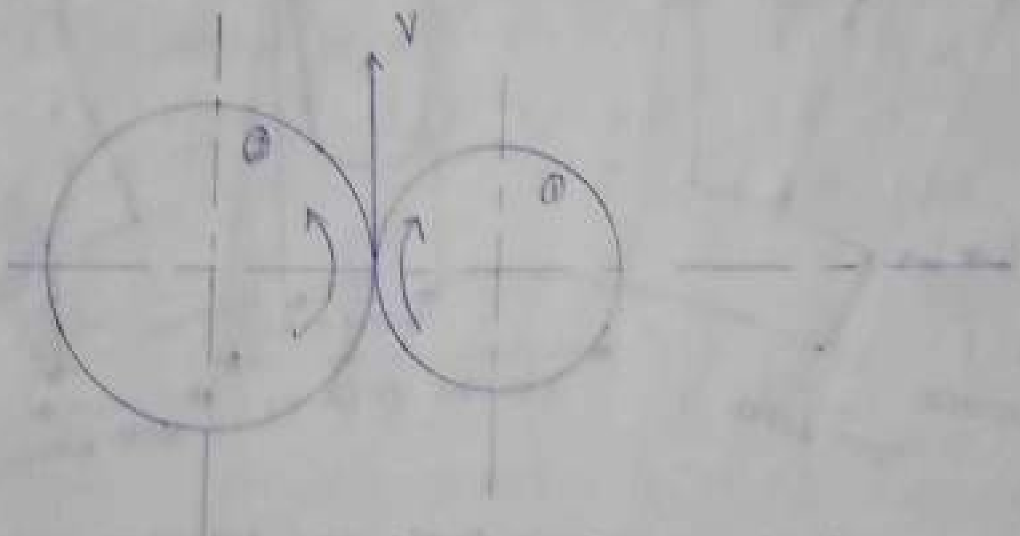
$= 39.39\ KW.$ (Ans)

Gears

The motion from one shaft to another shaft may be transmitted with belts, ropes and chains. These methods are mostly used when the two shafts are having long center distance. But if the distance between two shafts is very small, then gears are used to transmit motion froms one shaft to another.

The gear is defined as a toothed element which is used for transmitting rotary motion from one shaft to another.

In case of belts and ropes, the drive is not positive. There is slip and creep which reduces velocity ratio. But gear drive is a positive and smooth drive which transmits exact velocity ratio.



For no slip of the two surfaces, their tangential velocities of the contact surfaces should be same.

$$V_1 = V_2$$

$$\Rightarrow \omega_1 r_1 = \omega r_2$$

$$\Rightarrow \frac{2\pi N_1}{60} \times r_1 = \frac{2\pi N_2}{60} \times r_2$$

$$\Rightarrow N_1 \times r_1 = N_2 \times r_2$$

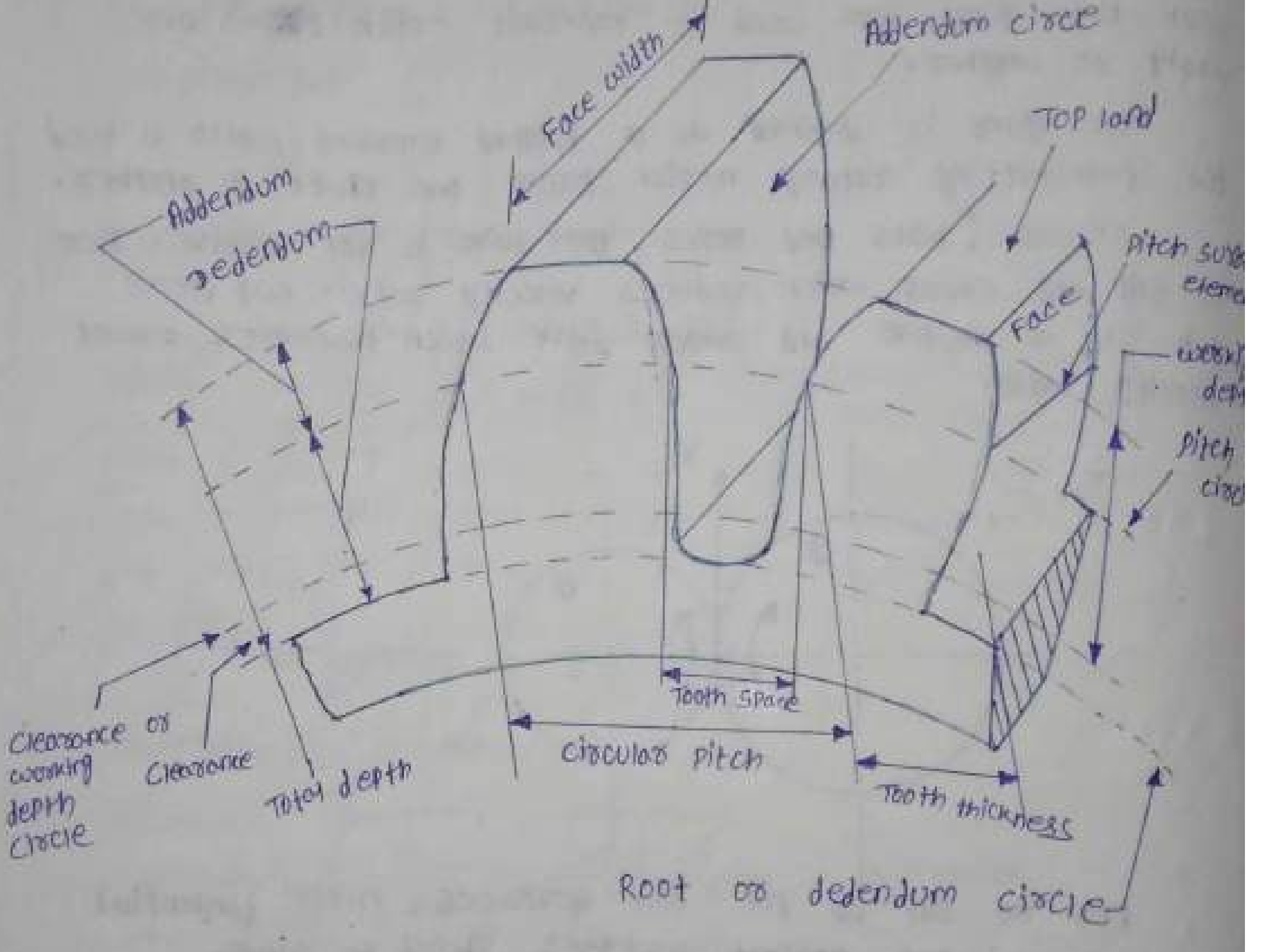
$$\boxed{\Rightarrow \frac{N_1}{N_2} = \frac{r_2}{r_1} = \frac{d_2}{d_1}}$$

Where, N_1 = speed of friction wheel 1 in rpm

N_2 = speed of friction wheel 2 in rpm

r_1 = Radius of wheel 1

r_2 = Radius of wheel 2



1. Pitch circle diameter / pitch diameter :-

It is the diameter of a circle which by pure rolling action would produce the same motion as the toothed gear wheel.

2. Pitch point :-

It is the point of contact of two pitch circles of mating gears.

3. Circular pitch (P_c) :-

It is the distance measured along the circumference of the pitch circle from a point on one tooth to a corresponding point on adjacent tooth.

4. Diametral pitch (P_d) :-

It is equal to the number of teeth per unit length of pitch circle diameter.

$$\boxed{P_d = \frac{T}{D}}$$

$$\boxed{P_c \times P_d = \pi}$$

5. Module (m) :-

It is defined as the length of the pitch circle diameter per tooth. It is denoted by 'm'.

$$\boxed{m = \frac{D}{T}}$$

$$\text{Module} = \frac{1}{\text{diametral pitch}}$$

$$P_c = \frac{\pi D}{T} = \pi m.$$

6. Addendum :-

It is the radial distance of tooth above pitch circle.

Addendum = 1 module.

7. Dedendum :-

It is the radial distance of tooth below the pitch circle.

Dedendum = 1.157 module $= \left(1 + \frac{\pi}{120}\right)$ module.

8. Addendum circle :-

It is the circle which passes through the top of teeth.

Diameter of Addendum circle = p.c.d + 2m

9. Dedendum circle :-

It is the circle which passes through the bottom of teeth.

Diameter of dedendum circle = p.c.d - 2 x 1.157m

10. Face of the tooth :-

It is that part of tooth surface which is above the pitch circle.

11. Flank of tooth :-

It is that part of tooth surface which is below the pitch circle.

12. Clearance :-

The radial height difference between the addendum and dedendum is known as clearance.

clearance = 1.157m - m = 0.157m.

13. profile :-

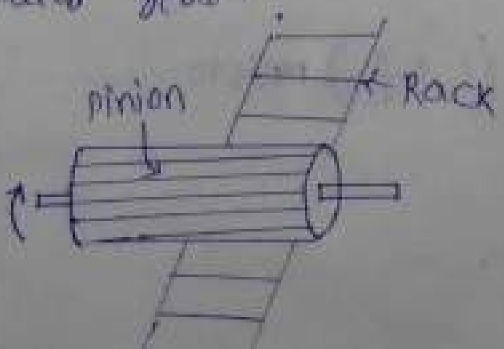
The circle forming face and flank is known as profile.

14. Rack :-

A gear wheel of infinite diameter is known as rack.

15. pinion :-

It is the smaller and usually the driving gear of a pair of mated gear.



27.04.23

Gear Trains

Introduction:-

Sometimes, two or more gears are made to mesh with each other to transmit power from one shaft to another. Such a combination is called gear train or train of toothed wheels. The nature of the train used depends upon the velocity ratio required and the relative position of the axes of shafts. A gear train may consist of spur, bevel or spiral gears.

Types of Gear Trains:-

Following are the different types of gear trains, depending upon the arrangement wheels.

1. Simple gear train
2. Compound gear train
3. Reverted gear train
4. Epicyclic gear train

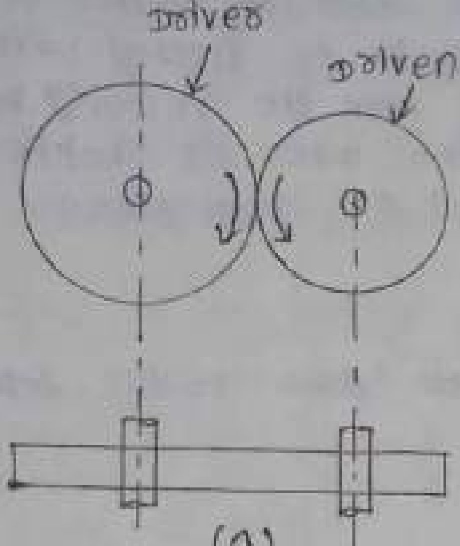
In the first three types of gear trains, the axes of the shafts over which the gears are mounted are fixed relative to each other. But in case of epicyclic gear trains, the axes of the shafts on which the gears are mounted may move relative to a fixed axis.

1. Simple gear train:-

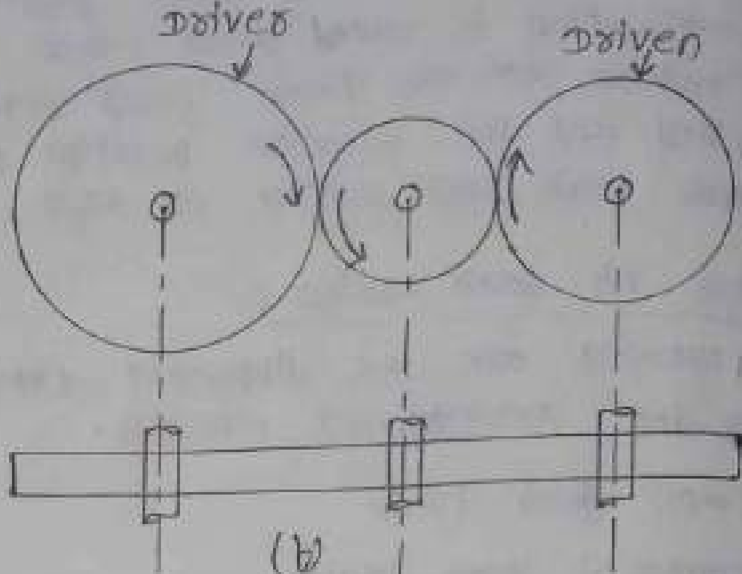
When there is only one gear on each shaft, as shown in fig., it is known as simple gear train. The gears are represented by their pitch circles.

When the distance between two shafts is small, the two gears 1 and 2 are made to mesh with each other to transmit motion from one shaft to other as shown in figure (a).

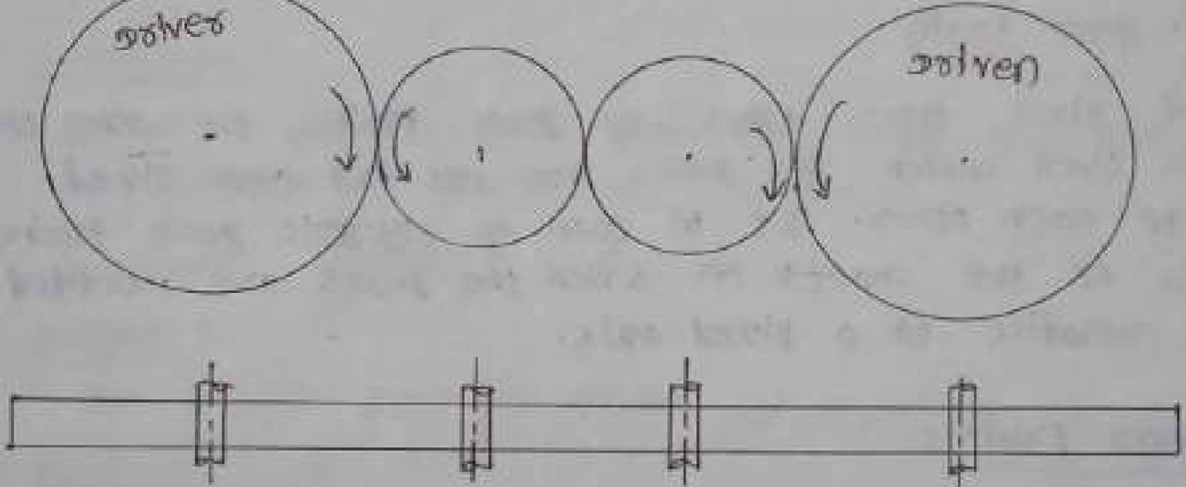
since the gear 1 drives the gear 2, therefore gear 1 is called driver and the gear 2 is called the driven or follower. It may be noted that the motion of the driven gear is opposite to the motion of driving gear.



(a)



(b)



Let N_1 = speed of gear 1 in r.p.m.

N_2 = speed of gear 2 in r.p.m.

T_1 = Number of teeth on gear 1.

T_2 = Number of teeth on gear 2.

since the speed ratio or velocity ratio of gear train is the ratio of the speed of the driver to the speed of the driven ~~number of teeth, these~~ or follower and ratio of speeds of any pair of gears in mesh is the inverse of their number of teeth, therefore

$$\text{speed ratio} = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

It may be noted that ratio of the speed of the driven or follower to the speed of the driver is known as train value of the gear train. Mathematically,

$$\text{Train value} = \frac{N_2}{N_1} = \frac{T_1}{T_2}$$

From the above, we see that the train value is the reciprocal of speed ratio.

sometimes, the distance between the two gears is large. The motion from one gear to another, in such a case, may be transmitted by either of the following two methods:

1. By providing the large sized gear.
2. By providing one or more intermediate gears.

A little consideration will show that the former method (i.e providing large sized gears) is very inconvenient and uneconomical method; whereas the latter method (i.e providing one or more intermediate gear) is very convenient and economical.

It may be noted that when the number of intermediate gears are odd. the motion of both the gears is like as shown in fig (b).

But if the number of intermediate gear are even, the motion of the driven or follower will be in the opposite direction of the driver as shown in fig (c).

Now consider a simple ~~ratio~~ train of gears with one intermediate gear as shown in fig. (b).

Let N_1 = speed of driver in r.p.m

N_2 = speed of intermediate gear in r.p.m

N_3 = speed of driven in r.p.m

T_1 = Number of teeth on driver

T_2 = Number of teeth on intermediate gear

T_3 = Number of teeth on driven.

Speed ratio ~~from~~ gear 1 and gear 2

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \quad \text{- - - - (i)}$$

Speed ratio ~~from~~ gear 2 to gear 3

$$\frac{N_2}{N_3} = \frac{T_3}{T_2} \quad \text{- - - - (ii)}$$

The speed ratio of the gear train as shown in fig (b) is obtained by multiplying the equations (i) and (ii)

$$\frac{N_1}{\cancel{N_2}} \times \frac{\cancel{N_2}}{N_3} = \frac{\cancel{T_2}}{T_1} \times \frac{T_3}{\cancel{T_2}}$$

$$\Rightarrow \frac{N_1}{N_3} = \frac{T_3}{T_1}$$

i.e speed ratio = $\frac{\text{speed of driver}}{\text{speed of driven}} = \frac{\text{No. of teeth on driven}}{\text{No. of teeth on driver}}$

Train value = $\frac{N_3}{N_1} = \frac{T_1}{T_3}$

$= \frac{\text{speed of driven}}{\text{speed of driver}} = \frac{\text{No. of teeth on driver}}{\text{No. of teeth on driven}}$

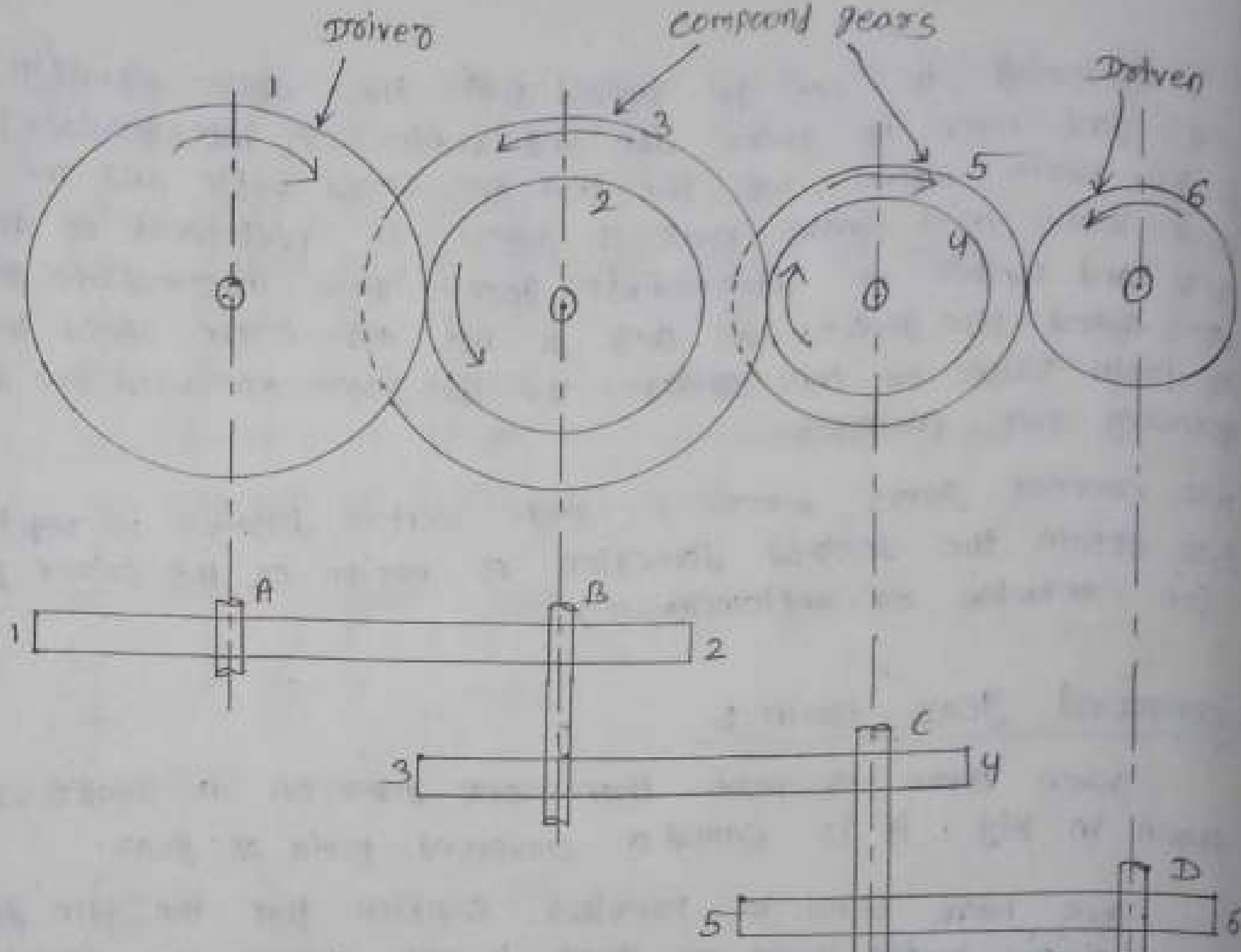
Similarly, it can be proved that the above equation holds good even if there are any number of intermediate gears. From above, we see that the speed ratio and the train value, in a simple train of gears, is independent of the size and number of intermediate gears. These intermediate gears are called idle gears, as they do not affect the speed ratio or train value of the system. The idle gears are used for the following two purposes.

1. To connect gears where a large centre distance is required.
2. To obtain the desired direction of motion of the driven gear. (i.e clockwise or anticlockwise).

2. Compound gear train :-

When there is more than one gear on a shaft, as shown in fig. it is called a compound train of gears.

We have seen in previous section that the idle gears, in a simple ~~ratio~~ train of gears do not affect the speed ratio of the system. But these gears are useful in bridging over the space between the driver and the driven. But whenever the distance between the driver and the driven or follower has to be bridged over by intermediate gears and at the same time a great speed ratio is required, then the advantage of intermediate gears is intensified by providing compound gears on intermediate shafts. In this case, each intermediate shaft has two gears rigidly fixed to it so that they may have the same speed. One of these two gears meshes with the driver and the other with the driven or follower attached to the next shaft as shown in fig..



In a compound gear train, as shown in fig. the gear 1 is the driving gear mounted on shaft A, gears 2 and 3 are compound gears which are mounted on shaft B. The gears 4 and 5 are also compound gears which are mounted on shaft C and the gear 6 is the driven gear mounted on shaft D.

Let, N_1 = speed of driving gear 1

T_1 = Number of teeth on driving gear 1

N_2, N_3, N_4, N_5, N_6 = speed of respective gears in rpm.

T_2, T_3, T_4, T_5, T_6 = Number of teeth on respective gears

since gear 1 is in mesh with gear 2, therefore its speed ratio is

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \quad \text{-----} (1)$$

similarly, for gears 3 and 4, speed ratio is

$$\frac{N_3}{N_4} = \frac{T_4}{T_3} \quad \text{------ (ii)}$$

similarly, for gears 5 and 6, speed ratio is

$$\frac{N_5}{N_6} = \frac{T_6}{T_5} \quad \text{----- (iii)}$$

The speed ratio of compound gear train is obtained by multiplying the equations (i), (ii) and (iii)

$$\frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5}$$

since gears 2 and 3 are mounted on one shaft B, therefore $N_2 = N_3$, similarly gears 4 and 5 are mounted on shaft C, therefore $N_4 = N_5$.

$$\frac{N_1}{N_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}$$

ie

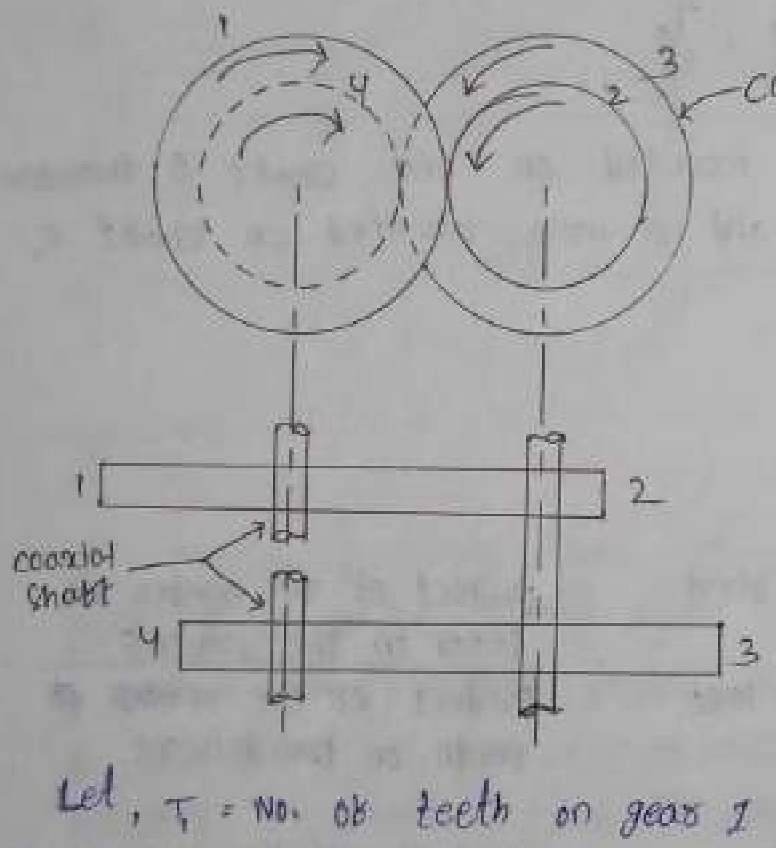
$$\text{speed ratio} = \frac{\text{speed of the first driver}}{\text{speed of the last driven}} = \frac{\text{product of the number of teeth on the drivens}}{\text{product of the number of teeth on the drivers}}$$

$$\text{Train ratio} = \frac{\text{speed of last driven}}{\text{speed of the first driver}} = \frac{\text{product of the number of teeth on the drivers}}{\text{product of the number of teeth on the drivens}}$$

The advantage of a compound train over a simple gear train is that a much larger speed reduction from the first shaft to the last shaft can be obtained with small gears. If a simple gear train is used to give a large speed reduction, the last gear has to be very large. Usually for a speed reduction in excess of 7 to 1, a simple train is not used and a compound train or worm gearing is employed.

3. Reverted Gear train :-

When the axes of 1st gear (i.e first driver) and last gear (i.e last driven) are coaxial then the gear train is known as reverted gear train.



Let, T_1 = No. of teeth on gear 1

R_1 = pitch circle radius of gear 1

N_1 = speed of gear 1 in rpm

similarly,

T_2, T_3, T_4 = No of teeth on respective gears

R_2, R_3, R_4 = pitch circle radius of respective gears

N_2, N_3, N_4 = speed of respective gears in r.p.m.

$$\boxed{R_1 + R_2 = R_3 + R_4}$$

The circular pitch or module of on the gears is assumed to be same, therefore number of teeth on each gear or directly proportional to its circumference or radius.

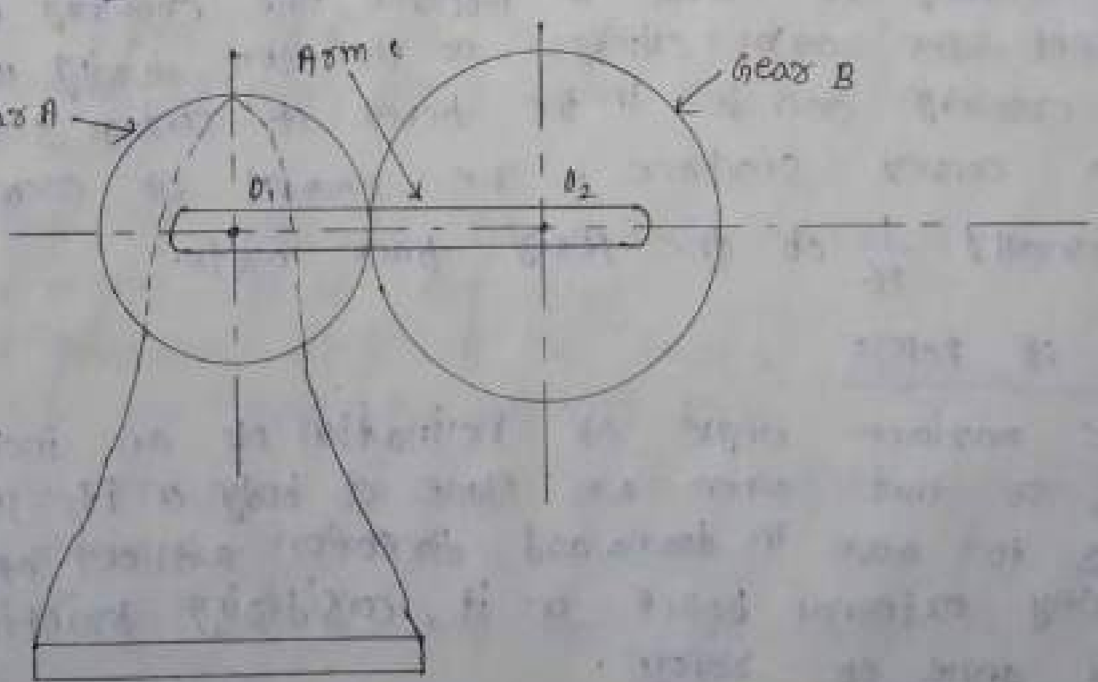
$$\boxed{T_1 + T_2 = T_3 + T_4}$$

speed ratio = $\dfrac{\text{product of no of teeth on driven}}{\text{product of no of teeth on driver}}$

$$\boxed{\frac{N_1}{N_4} = \frac{T_2 \times T_4}{T_1 \times T_3}}$$

4. Epicyclic gear train :-

In an epicyclic gear train, the axes of ~~moved relative~~ the shafts, over which the gear are mounted, may moved relative to fixed axis. A simple epicyclic gear ~~train~~ train is shown in fig. below. Where 'a gear 'A' and arm 'C' have a common axis at O_1 about which they can rotate. The gear B meshes with gear A and its axis on the arm at O_2, about which the gear B can rotate.



If the arm is fixed, the gear train is simple and gear A can drive gear B or vice versa, but if gear A is fixed and the arm is rotated about the axis of gear A (i.e O_1), then the gear B is forced to rotate upon and around gear A. Such a motion is called epicyclic and gear trains are arranged in such a manner that one or more of their members move up on and around another member are known as epicyclick gear train. (Epi means upon and cyclic means around). The epicyclic gear trains may be simple or compound.

The epicyclic gear trains are useful for transmitting high velocity ratios with gears of moderate size on a comparatevely space. The epicyclic gear trains are used in differential gears in automobile, wrist watches etc.

① Crowning of pulley.

The slight convexity of pulley rims is known as crowning. It is made so to keep the belt in centre on a pulley rim while in motion. The crowning also prevent the axial slipping of the belt during operation. The crowning can be in the form of conical surface or a convex surface. The amount of crowning is usually $\frac{1}{96}$ of the pulley face width.

② Angle of Repose

The maximum angle of inclination of an inclined plane, so that when we place a body on it, just starts to move in downward direction without application of any external force on it, considering friction is called angle of repose.

UNIT - 4

2.5.23

Governor and Flywheels

Governor

Governor is a device used for maintaing a constant mean speed of rotation of the crank shaft over a long period of time during which the load on the engine may be varry.

When the load on the engine increases the speed of the engine will be decreases.

The governor will act in such a way that it will increases the supply of working fluid.

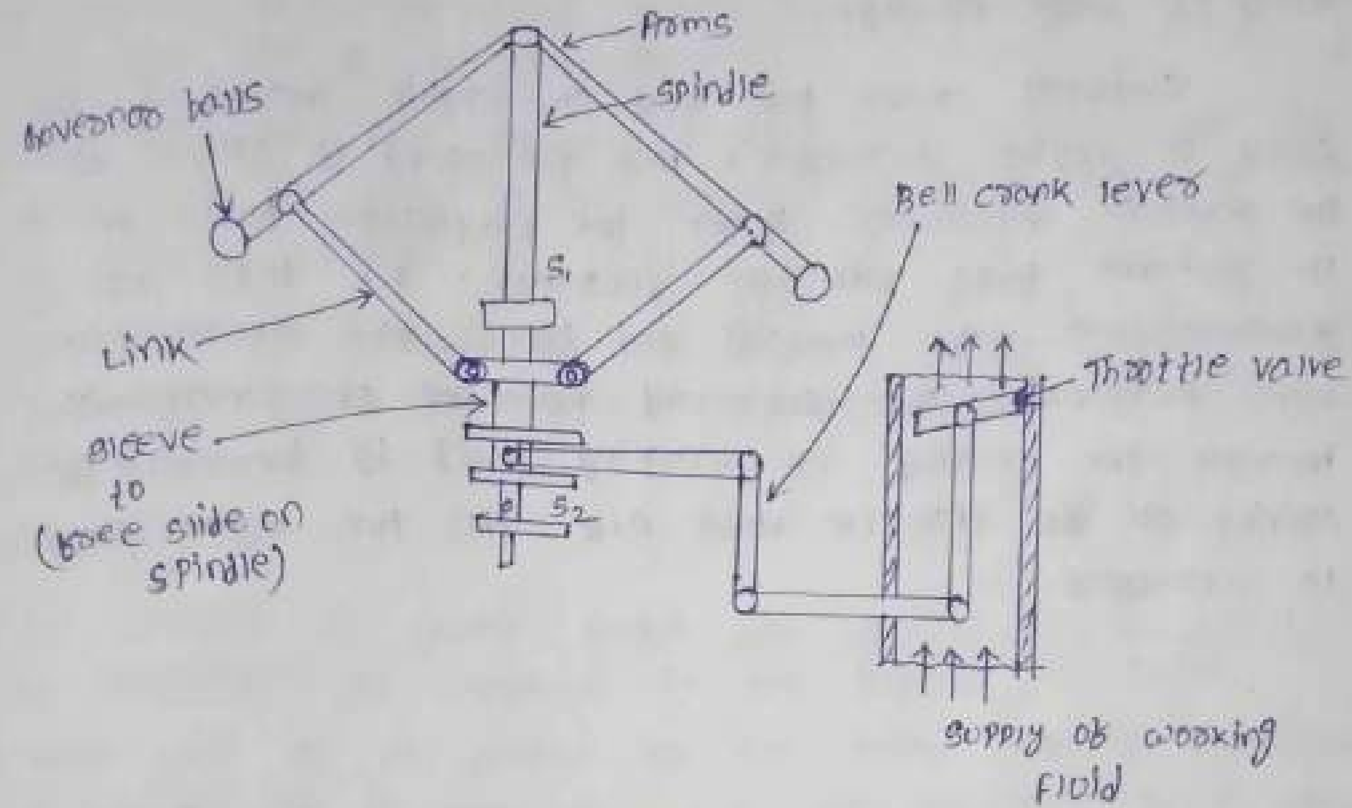
Similarly when the load on the engine decreases, the speed of the engine will be increases. Then the governor will act in such a way that the supply of working fluid decreases.

Thus the main speed of rotation will be maintain constant as closely as possible over a long period of time.

Classification of Governor

- Governor
  - Centrifugal governor
    - Pendulum type
      - ✓ Watt governor
    - Loaded type
      - Dead weight governor
        - ✓ Porter governor
        - ✓ Proell governor
      - Spring controlled governor
        - ✓ Hartnell governor
        - Hartung governor
        - Pickering governor
        - Wilson-Hartnell governor
  - Inertia governor

Centrifugal Governor



It consists of two balls of equal masses attached to the two arms. The upper ends of arms are pivoted to a spindle, which is driven by engine through bevel gears. The lower arms are connected to a sleeve. which is keyed to spindle. The sleeve pivot with spindle but can slide up and down. The two stops S_1 and S_2 on the spindle prevents upward and downward motion of sleeve. The sleeve is connected by a bell crank lever to a throttle valve which controls the supply of working fluid. When the sleeve rises, the supply of working fluid decreases and when sleeve falls, the supply of working fluid increases.

When the load on the engine decreases, the speed of engine increases. As the spindle of governor is driven by engine, hence the speed of spindle also increases. This will increase the centrifugal force 'm' the governor balls ~~outwards~~, and the balls will move outwards. Due to movement of balls outwards, the sleeve will rise upward. The upward movement of the sleeve will operate a throttle valve at the other end of bell crank lever to reduce the supply of working fluid or by reducing the supply of valve opening.

Similarly when the load on engine increases, the speed of engine decreases. Also the speed of spindle of the governor decreases. Hence the centrifugal force on the governor balls will also decrease. The balls of governor will move inwards and hence the sleeve will move downwards. The downward movement of sleeve will increase the supply of working fluid by increasing the opening of the throttle valve and. thus the engine speed is increased.

Watt Governor

→ It is the simplest form of centrifugal governor.

→ It consists of two balls which are attached to the spindle with the help of links or arms.

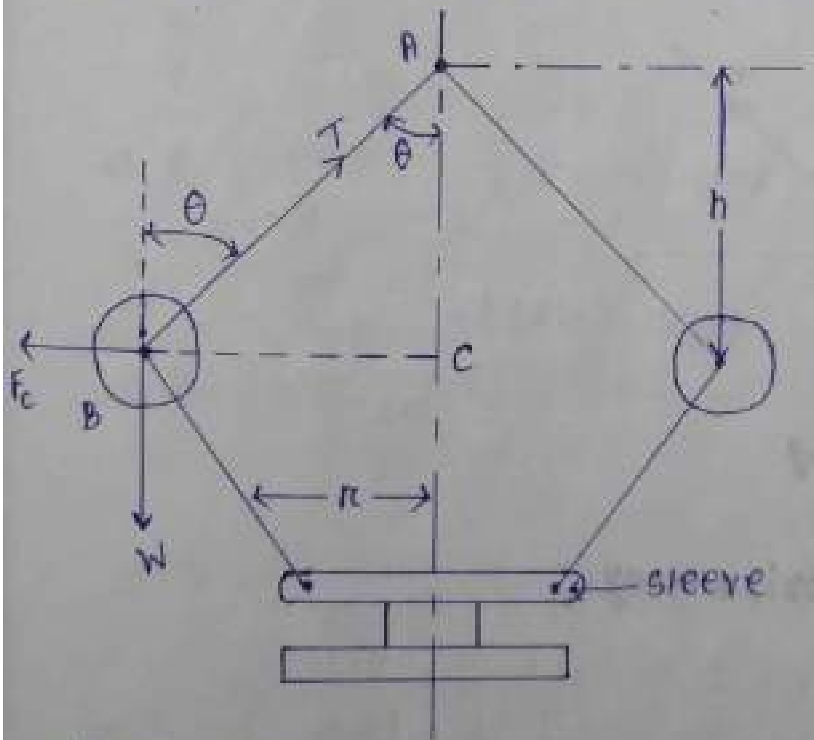
→ The upper link are pinned at point 'O', where are the lower link are fixed to the sleeve. which is free to move on vertical spindle.

→ The spindle is driven by the engine, As the spindle rotates, the balls takes up a position depending upon the speed of spindle.

Assumption:-

→ The weight of arms, links and sleeve is assumed to be negligible as compare to the weight of balls.

→ There will be no tension in the lower link if the sleeve is assume to be mass less and also the friction is neglected



Let, m = Mass of each ball

W = weight of each ball = mg

T = Tension in the arm

ω = Angular velocity of the ball

r = Radial distance of the ball centre from the spindle axis.

F_c = centrifugal force acting in the ball

$= \frac{mv^2}{r} = \frac{m(\omega \times r)^2}{r} = \frac{m\omega^2 r^2}{r} = m\omega^2 r.$

h = height of governor i.e the vertical distance from the centre of ball to the point of intersection of the upper arms along the axis of spindle.

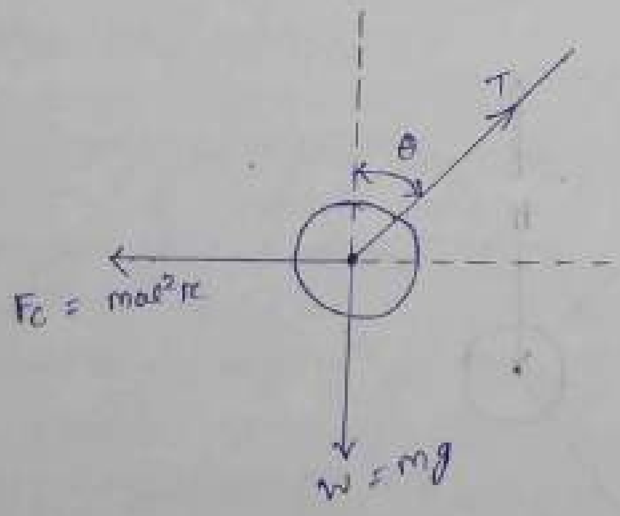
Each ball will be in equilibrium under the action of following three process.

(i) centrifugal force acting on the ball i.e

$F_c = m\omega^2 r$

(ii) ~~Tension in the~~ weight of ball i.e $W = mg$

(iii) Tension in the upper arm i.e T



Resolving all forces horizontally

$\Sigma H = 0$

$F_c = T \sin\theta$

$\Rightarrow T \sin\theta = m\omega^2 r$ ------ (i)

Resolving all forces vertically

$\Sigma V = 0$

$W = T\cos\theta$

$\Rightarrow T\cos\theta = mg$ ------ (ii)

Dividing eqn (i) and (ii), we get

$$\Rightarrow \frac{\cancel{T}\sin\theta}{\cancel{T}\cos\theta} = \frac{\cancel{m}\omega^2 r}{\cancel{m}g}$$

$$\Rightarrow \tan\theta = \frac{\omega^2 r}{g} \text{ ------ (iii)}$$

But we know that, in ΔABC

$$\tan\theta = \frac{BC}{AC} = \frac{r}{h} \text{ ------ (iv)}$$

From eqn (iii) & (iv) we get

$$\Rightarrow \frac{\cancel{r}}{h} = \frac{\omega^2 \cancel{r}}{g}$$

$$\Rightarrow g = \omega^2 h$$

$$\Rightarrow h = \frac{g}{\omega^2}$$

$$\Rightarrow h = \frac{g}{\left(\frac{2\pi N}{60}\right)^2}$$

$$\Rightarrow h = \frac{g}{\left(\frac{2\pi}{60}\right)^2} \times \frac{1}{N^2}$$

$$\Rightarrow h = \frac{9.81}{\left(\frac{2\pi}{60}\right)^2} \times \frac{1}{N^2}$$

$$\boxed{\Rightarrow h = \frac{894.56}{N^2} = \frac{895}{N^2}.}$$

HW

① calculate change in vertical height of a watt governor when its speed increase from 50 rpm to 51 rpm, increase from 200 rpm to 201 rpm.

Ans: Given data,

(i) $N_1 = 50$ rpm

$N_2 = 51$ rpm

$$h_1 = \frac{895}{(N_1)^2} = \frac{895}{(50)^2} = 0.358$$

$$h_2 = \frac{895}{(N_2)^2} = \frac{895}{(51)^2} = 0.344$$

$$h = h_1 - h_2$$

$$= 0.358 - 0.344 = 0.014m$$

(ii) $N_1 = 200$ rpm

$N_2 = 201$ rpm

$$h_1 = \frac{895}{(N_1)^2} = \frac{895}{(200)^2} = \cancel{0.02m}\ 0.0895\ m$$

$$h_2 = \frac{895}{(N_2)^2} = \frac{895}{(201)^2} = \cancel{0.22}\ 0.0877\ m$$

$$h = h_1 - h_2$$

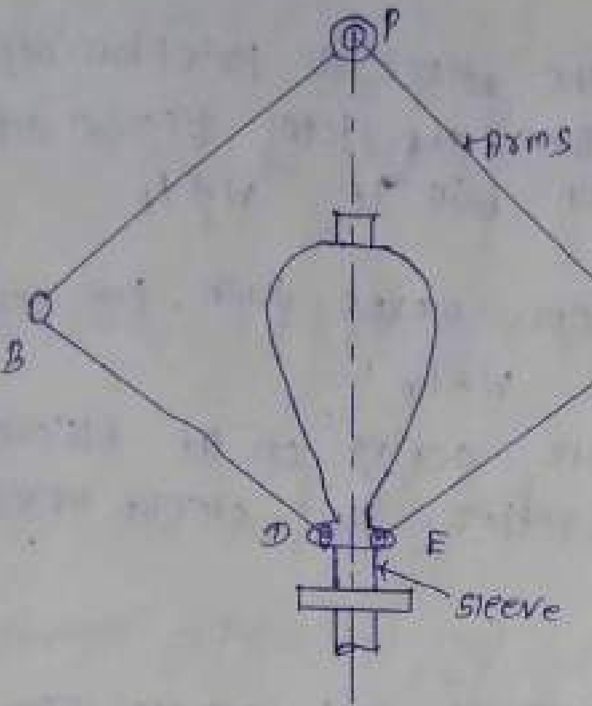
$$= 0.0895 - 0.0877$$

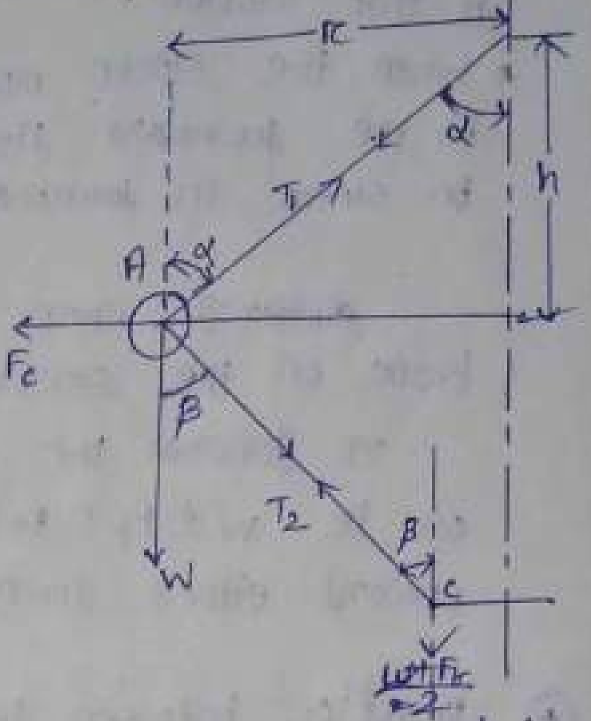
$$= 1.8 \times 10^{-3}\ m.$$

$$= 0.0018\ m.$$

3.5.2023

Porter Governor





Incase of porter governor, a central heavy load is attached to the sleeve. The central load and sleeve moves up and down.

Let, M = mass of central load

W = weight of central load = Mg

m = mass of each ball

w = weight of each ball

h = height of the governor

r = Radius of rotation

F_c = centrifugal force on ball = $m\omega^2 r$

ω = angular speed of ball = $2\pi N/60$

N = speed of ball in rpm

T_1 = Tension in the upper arm

T_2 = Tension in the lower arm or link

α = angle of inclination of the upper arm to vertically.

β = angle of inclination of the lower arm to vertically

F_f = Frictional force between sleeve and spindle

$= \frac{W \pm F_f}{2}$

The force of friction always acts in a direction oppo to that motion.

* When the sleeve moves up the force of friction acts in the downward direction. Then the total force acting on sleeve in downward direction will be $W + F_f$

Similarly when the sleeve moves down, the total force on the sleeve will be $W - F_f$.

In general the total force acting on the sleeve will be $W \pm F_f$. Depending whether the sleeve moves upward either downward.

(*) Relation between height of governor and angular speed of ball (h, ω) :-

Equilibrium of left ball which is acted upon by

(i) centrifugal force (F_c)

(ii) weight

(iii) Tension in upper arm (T_1)

(iv) Tension in lower link (T_2)

considering the equilibrium of the left half of sleeve we have

$$\Rightarrow T_2 \cos\beta = \frac{W \pm F_f}{2}$$

$$\Rightarrow T_2 = \frac{W \pm F_f}{2\cos\beta} \quad \text{------ (i)}$$

Ball

Again $\Sigma V = 0$

$$\Rightarrow T_1 \cos\alpha = w + T_2 \cos\beta \quad \text{------ (ii)}$$

$\Sigma H = 0$

$$\Rightarrow F_c = T_1 \sin\alpha + T_2 \sin\beta \quad \text{---- (iii)}$$

from eq^n (i) and (iii) we get

$$\Rightarrow F_c = T_1 \sin\alpha + \left(\frac{W \pm F_r}{2\cos\beta}\right) \times \sin\beta$$

$$\Rightarrow T_1 \sin\alpha = F_c - \left(\frac{W \pm F_r}{2\cos\beta}\right) \times \sin\beta$$

$$\Rightarrow T_1 \sin\alpha = F_c - \left(\frac{W \pm F_r}{2}\right) \times \frac{\sin\beta}{\cos\beta}$$

$$\Rightarrow T_1 \sin\alpha = F_c - \left(\frac{W \pm F_r}{2}\right) \times \tan\beta \quad \text{------- (iv)}$$

Deviding eq^n (iv) and (ii) ; we get

$$\Rightarrow \frac{eq^n\,(iv)}{eq^n\,(ii)} \Rightarrow \frac{\cancel{T_1} \sin\alpha}{\cancel{T_1} \cos\alpha} = \frac{F_c - \left(\frac{W \pm F_r}{2}\right) \times \tan\beta}{w + T_2 \cos\beta}$$

$$\Rightarrow \tan\alpha = \frac{F_c - \left(\frac{W \pm F_r}{2}\right) \tan\beta}{w + T_2 \cos\beta}$$

$$\Rightarrow \tan\alpha = \frac{F_c - \left(\frac{W \pm F_r}{2}\right) \tan\beta}{w + \left(\frac{W \pm F_r}{2}\right)} \quad (\because \text{From eq}^n \text{ (i)})$$

$$\Rightarrow \tan\alpha \left\{ w + \left(\frac{W \pm F_r}{2}\right) \right\} = F_c - \left(\frac{W \pm F_r}{2}\right) \tan\beta$$

$$\Rightarrow w + \left(\frac{W \pm F_r}{2}\right) = \frac{F_c}{\tan\alpha} - \left(\frac{W \pm F_r}{2}\right)\left(\frac{\tan\beta}{\tan\alpha}\right)$$

In Δ OAB $\tan\alpha = \frac{AB}{OB} = \frac{r_c}{h}$

Let $\frac{\tan\beta}{\tan\alpha} = K$

$$\Rightarrow w + \left(\frac{W \pm F_r}{2}\right) = \frac{F_c}{r/h} - \left(\frac{W \pm F_r}{2}\right) k$$

$$\Rightarrow w + \left(\frac{W \pm F_r}{2}\right) = \frac{m\omega^2 r}{r/h} - \left(\frac{W \pm F_r}{2}\right) k$$

$$\Rightarrow w + \left(\frac{W \pm F_r}{2}\right) = m\omega^2 h - \left(\frac{W \pm F_r}{2}\right) k$$

$$\Rightarrow m\omega^2 h = w + \left(\frac{W \pm F_r}{2}\right) + \left(\frac{W \pm F_r}{2}\right) k$$

$$\Rightarrow m\omega^2 h = mg + \left(\frac{W \pm F_r}{2}\right)(1+k)$$

$$\Rightarrow \omega^2 = \frac{mg + \left(\frac{Mg \pm F_r}{2}\right)(1+k)}{mh} \quad \text{------ (v)}$$

$$\Rightarrow \omega^2 = \frac{mg + \left(\frac{Mg \pm F_r}{2}\right)(1+k)}{m} \times \frac{1}{h}$$

$$\Rightarrow \omega^2 h = \frac{w + \left(\frac{W \pm F_r}{2}\right)(1+k)}{m}$$

$$\Rightarrow h = \frac{w + \left(\frac{W \pm F_r}{2}\right)(1+k)}{m\omega^2}$$

If there is no friction between sleeve and spindle then $F_r = 0$

$$\Rightarrow h = \frac{w + \frac{Mg}{2}(1+k)}{m\omega^2}$$ [in terms of weight]

{In terms of mass}

$$\Rightarrow h = \frac{mg + \frac{Mg}{2}(1+k)}{m\omega^2}$$

$$\Rightarrow h = \frac{g\left[m + \frac{M}{2}(1+k)\right]}{m\omega^2}$$

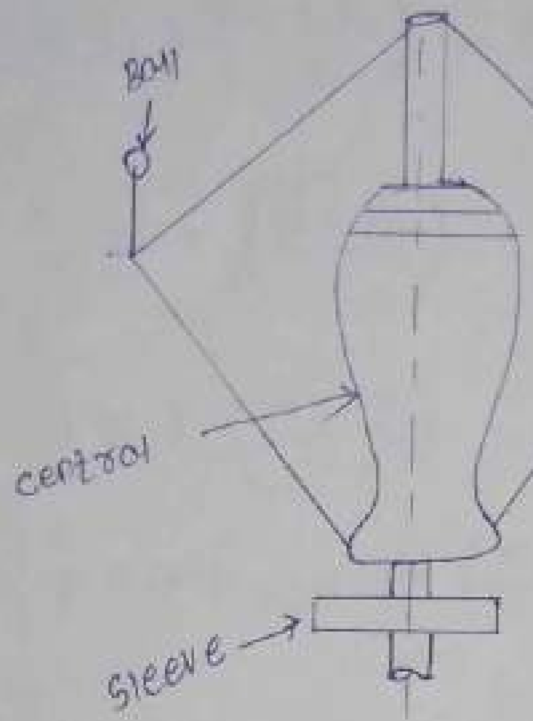
$$\Rightarrow \omega^2 = \frac{m + \frac{M}{2}(1+k)}{m} \times \frac{g}{h}$$

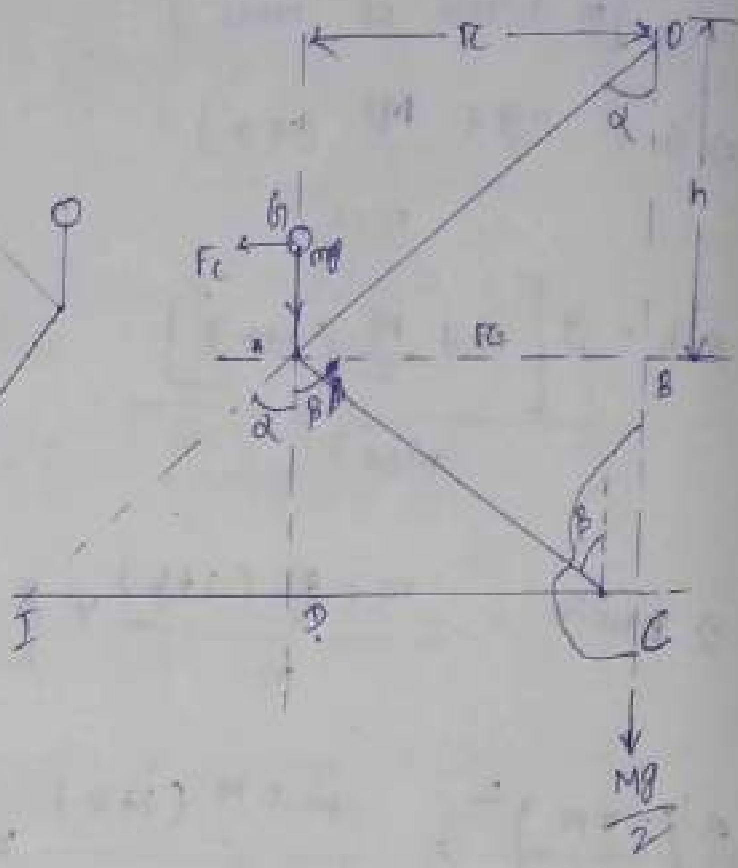
$$\Rightarrow \left(\frac{2\pi N}{60}\right)^2 = \frac{m + \frac{M}{2}(1+k)}{m} \times \frac{g}{h}$$

$$\Rightarrow N^2 = \frac{m + \frac{M}{2}(1+k)}{m} \times \frac{g}{h} \times \left(\frac{60}{2\pi}\right)^2$$

$$\Rightarrow N^2 = \frac{m + \frac{M}{2}(1+k)}{m} \times \frac{895}{h}.$$

Proell Governor





proowell Governor is similar to porter governor having a heavy central load at sleeve. But in different form porter governor in the arrangement balls.

The balls are carried on the extension of lower arms instead of at the junction of upper and lower arm.

The proowell governor may be analized by considering the equilibrium of lower arm AC.

The instantaneous centre I of the lower arm AC is obtained by producing line OA and by drawing a line through C perpendicular to governor axis.

Taking moment of all forces $\left(F_c, mg, \frac{Mg}{2}\right)$ above I and assuming the extension AG of lower arm to be vertical we get

$$F_c \times GD = mg \times ID + \frac{Mg}{2} \times IC$$

deviding both side AD we get.

$$\Rightarrow F_c \times \frac{GD}{AD} = mg \times \frac{ID}{AD} + \frac{Mg}{2} \times \frac{IC}{AD}$$

$$\Rightarrow F_c \times \frac{GD}{AD} = mg \times \tan\alpha + \frac{Mg}{2} + \left(\frac{ID + DC}{AD}\right) \quad [\because IC = ID + DC]$$

$$\Rightarrow F_c \times \frac{GD}{AD} = mg * \tan\alpha + \frac{Mg}{2} + \left[\frac{ID}{AD} + \frac{DC}{AD}\right]$$

$$= mg \times \tan\alpha + \frac{Mg}{2} + [\tan\alpha + \tan\beta]$$

$$= mg \times \tan\alpha + \frac{Mg}{2} \tan\alpha \left[1 + \frac{\tan\beta}{\tan\alpha}\right]$$

$$= mg \tan\alpha + \frac{Mg}{2} \tan\alpha [1 + K] \quad \left[\because \frac{\tan\beta}{\tan\alpha} = K\right]$$

$$\Rightarrow F_c \times \frac{GD}{AD} = \tan\alpha \left[mg + \frac{Mg}{2}(1+K)\right]$$

$$\Rightarrow F_c = \frac{AD}{GD} \times \tan\alpha \left[mg + \frac{Mg}{2}(1+K)\right]$$

in Δ AOB $\tan\alpha = \frac{r}{h}$

and $F_c = m\omega^2 r$

by putting above value we get.

$$\Rightarrow m\omega^2 r = \frac{AD}{GD} \times \frac{r}{h} \left[mg + \frac{Mg}{2}(1+K)\right]$$

$$\Rightarrow m\omega^2 = \frac{AD}{GD} \times \frac{1}{h} \left[mg + \frac{Mg}{2}(1+K)\right]$$

$$\Rightarrow \omega^2 = \frac{AD}{GD} \times \frac{1}{mh} \left[mg + \frac{Mg}{2}(1+K)\right]$$

$$\Rightarrow \omega^2 = \frac{AD}{GD} \times \left[\frac{mg + \frac{Mg}{2}(1+K)}{mh} \right]$$

If, $K = 1$,

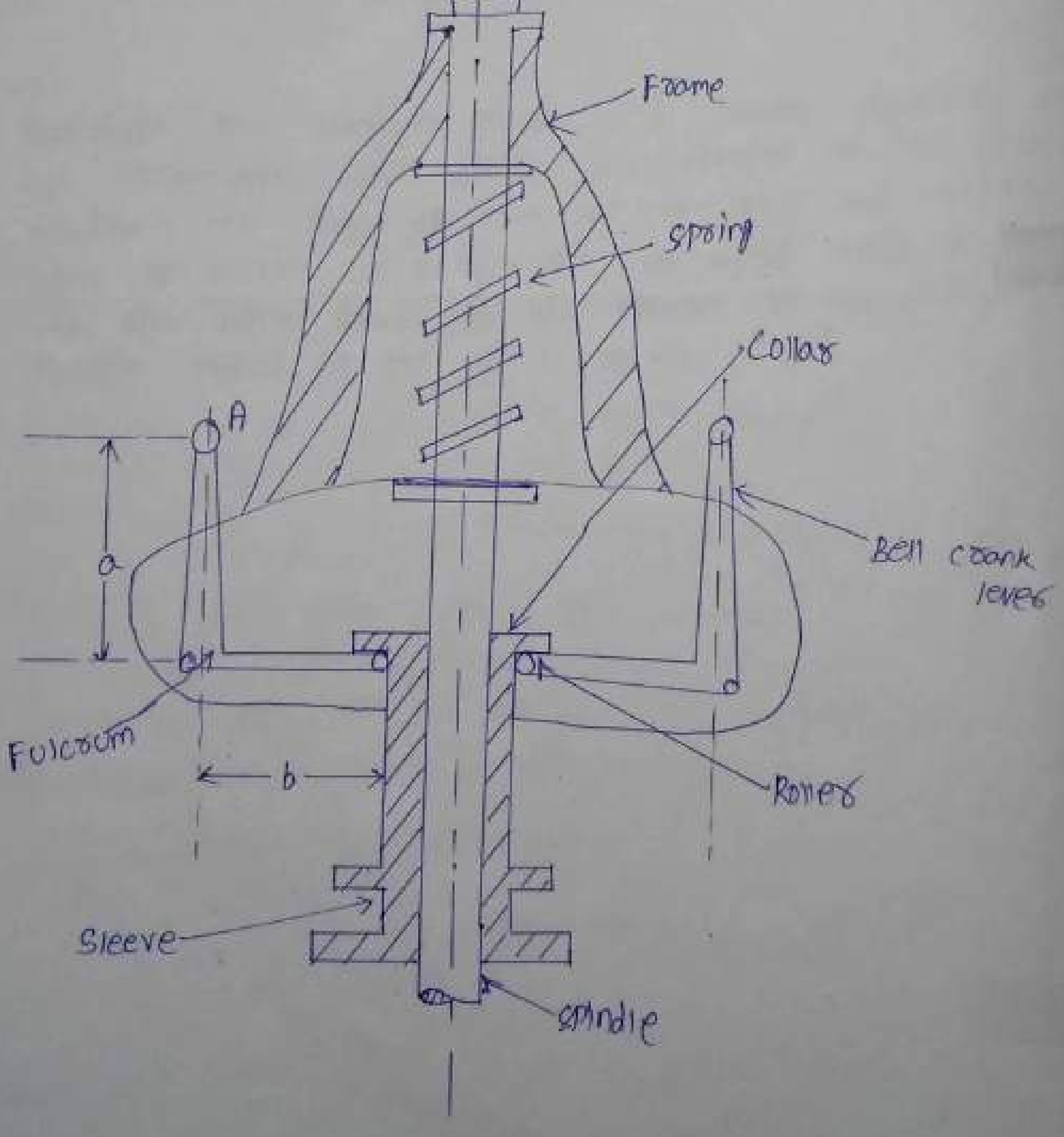
then $\Rightarrow \omega^2 = \frac{AD}{GD} \times \left[\frac{mg + \frac{Mg}{2}(1+1)}{mh} \right]$

$$\boxed{\Rightarrow \omega^2 = \frac{AD}{GD} \times \left[\frac{g(m+M)}{mh} \right].}$$

① Calculate the minimum speed of a proell governor which has equal arm each 200 mm and pivoted on the axis of rotation. The mass of each ball is 4 kg and central mass of sleeve is 20 kg. The extension arms of lower link are each 60 mm long and parallel to the axis when minimum radius of the ball is 100 mm.

Hartnell Governor

Hartnell governor is a spring loaded governor. Two bell crank levers each carrying a ball at one end and a roller at other are pivoted at point 'o' and 'o' to the frame. The roller fit in to a groove in the sleeve. The frame is attached to the governor spindle and hence rotate with it.



When the speed increases the radius of rotation of balls increases and the balls move away from spindle. The both are connected to bell crank levers which are pivoted at points 'o' and 'o''. As the both move away from spindle axis, the rollers (connected at other end of bell crank lever) lift the sleeve against the spring force. If the speed decreases the sleeve move downward. The movement of sleeve is to the throttle of engine to control the amount of energy supplied to the engine.

Let, r_1 = Min radius of rotation of ball centre from spindle axis.

r_2 = Maximum radius of rotation of ball centre from spindle arm.

S_1 = spring force exerted on sleeve at minimum radius.

S_2 = spring force exerted on sleeve at maximum radius.

m = Mass of each ball

M = Mass of sleeve

ω_1 and ω_2 = Minimum and maximum angular velocity.

N_1 = Minimum speed of governor at minimum radius.

N_2 = Maximum speed of governor at maximum radius.

F_{C_1} = centrifugal force on minimum radius. (of ball)

F_{C_2} = centrifugal force on maximum radius. (of ball)

s = stiffness of spring or force required to compress the spring by 1mm.

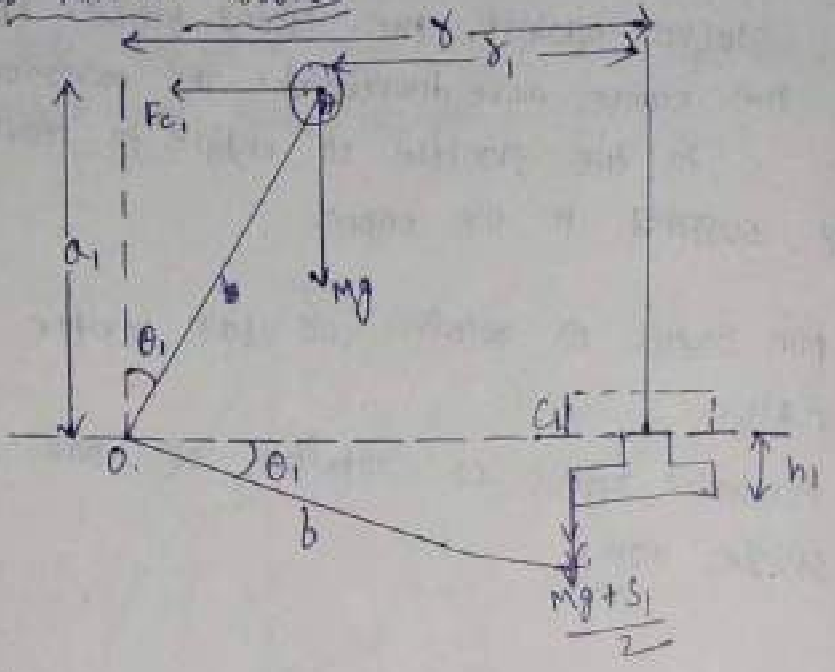
a = length of ball arm of bell crank lever i.e OA

b = length of sleeve arm on bell crank lever i.e OC.

r = Radius of rotation when the governor is in mid position

h = compression of the spring when radius of rotation change from r_1 to r_2. This is known as lift of sleeve.

Position of minimum radius



The position of bell crank lever at minimum radius is shown by AOC, where the position of bell crank lever, when the governor is in mid position is shown by dotted line A_1OC_1.

h_1 = lift of sleeve i.e vertical distance CC_1.

The angle turned by bell crank lever between mid position and minimum radius of position is θ_1. This means the angle between OA and OA_1 is same as between OC and OC_1.

$$\Rightarrow \sin\theta = \frac{CC_1}{OC} = \frac{AA_1}{OA}$$

$$\Rightarrow \frac{h_1}{b} = \frac{r - r_1}{b}$$

$$\Rightarrow h_1 = \frac{b(r - r_1)}{a} \quad \text{---(i)}$$

position of minimum radius :-

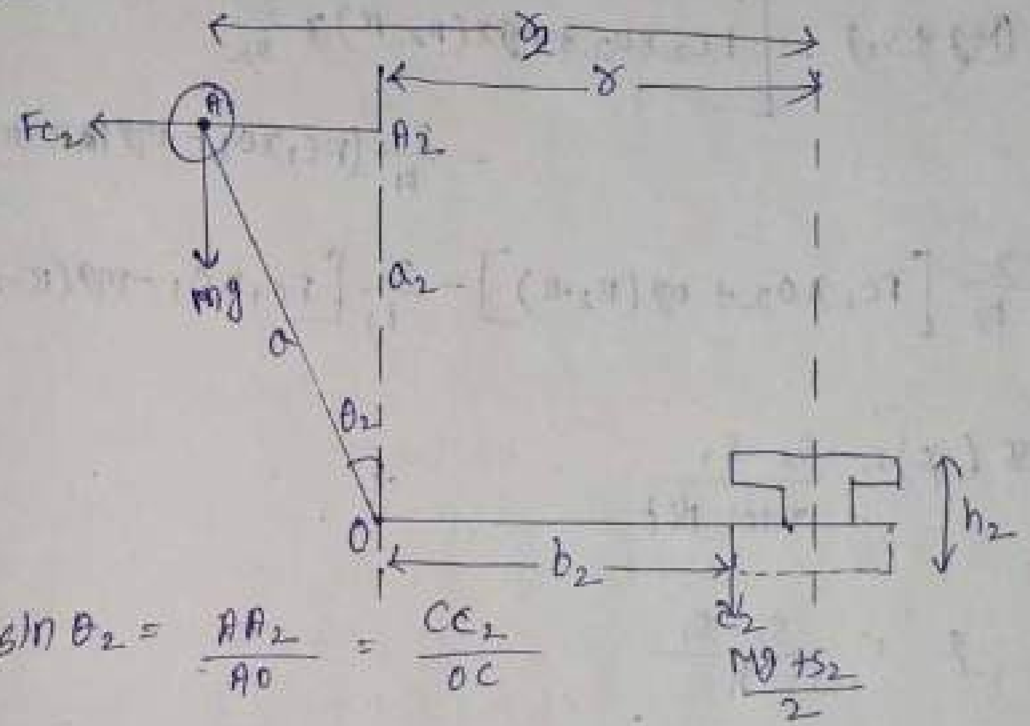
Taking moment of all forces about point O.

$\Rightarrow FC_1 \times a_1 = mg \times (r - r_1) + \frac{Mg + S_1}{2} \times b_1$

$\Rightarrow \frac{Mg + S_1}{2} \times b_1 = FC_1 \times a_1 - mg(r - r_1)$ ---- (ii)

$\Rightarrow Mg + S_1 = \frac{2}{b_1}\{FC_1 \times a_1 - mg(r - r_1)\}$ ---- (ii)

position of maximum radius



$\sin\theta_2 = \frac{AA_2}{AO} = \frac{CC_2}{OC}$

$\Rightarrow \frac{CC_2}{OC} = \frac{AA_2}{OA}$

$\Rightarrow \frac{h_2}{b} = \frac{r_2 - r}{a}$

$\Rightarrow h_2 = \frac{b(r_2 - r)}{a}$ ------ (iii)

Adding eqn (i) and (iii) we get

$h_1 + h_2 = \frac{b}{a}(r - r_1) + \frac{b}{a}(r_2 - r)$

$\Rightarrow h = \frac{b}{a}(r - r_1 + r_2 - r)$

$\Rightarrow h = \frac{b}{a}(r_2 - r_1)$

Taking moment of all forces about fulcrum O.

$$\Rightarrow FC_2 \times a_2 + mg \times (r_2 - r) = \frac{Mg + S_2}{2} \times b_2$$

$$\Rightarrow \frac{Mg + S_2}{2} = \{FC_2 \times a_2 + mg(r_2 - r)\} \times \frac{2}{b_2} \quad \text{---- (iv)}$$

substracting eq (iii) from eqn (iv) we get

$$\Rightarrow (Mg + S_2) - (Mg + S_1) = \Big[FC_2 \times a_2 + mg \times (r_2 - r) \times \frac{2}{b_2}$$

$$- \frac{2}{b_1}(FC_1 \times a_1 - mg(r - r_1))\Big]$$

$$\Rightarrow S_2 - S_1 = \frac{2}{b_2}\left[FC_2 \times a_2 + mg(r_2 - r)\right] - \frac{2}{b_1}\left[FC_1 \times a_1 - mg(r - r_1)\right]$$

spring stiffness $(s) = \frac{S_2 - S_1}{\text{total lift}}$

$$s = \frac{S_2 - S_1}{h}$$

① A Hartnell Governor having a central sleeve and right angled bell crank levers operates between 290 rpm and 310 rpm for a sleeve lift of 15 mm if the sleeve arms and arms are 80 mm and 120 mm respectively. The levers pivoted at 120 mm from the governor is and mass of each ball is 2.5 kg. The ball arms are parallel to the governor axis at the lowest equilibrium speed. Determine

(i) load on the spring at lowest and highest equilibrium speed.

(ii) Stiffness of spring.

Isochronism:-

A governor is said to be isochronous if the equilibrium speed is constant for a radii of relation of the balls within the working range. This means that when radius of rotation changes from minimum radius to maximum radius, the equilibrium speed remains constant.

r_1 = minimum radius of rotation

N_1 = Minimum speed

r_2 = Maximum radius of rotation

N_2 = Maximum speed

$$\boxed{\text{For isochronism} \Rightarrow N_1 = N_2}$$

Stability:-

A governor is said to be stable when for each speed there is only one radius of rotation of the governor balls at which the governor is in equilibrium. The speed should be within working range of governor. $\boxed{r_1 = r_2}$

Sensitiveness:-

Sensitiveness is defined as the ratio of difference between the maximum and minimum equilibrium speeds to the mean equilibrium speed

N_1 = minimum equilibrium speed

N_2 = Maximum equilibrium speed

N = Mean equilibrium speed

$= \frac{N_1 + N_2}{2}$.

sensitiveness = $\frac{\text{Difference bet}^n \text{ max}^m \text{ and min}^n \text{ equilibrium speed}}{\text{Mean equilibrium speed}}$

$$\Rightarrow \frac{N_2 - N_1}{N} = \frac{N_2 - N_1}{\frac{N_1 + N_2}{2}} = \frac{2(N_2 - N_1)}{N_1 + N_2}$$

Flywheels :-

A flywheel is used to control the speed variations caused by fluctuation of energy during each cycle of operation.
* It acts as a reservoir of energy which stores energy during the period when the supply of energy is more than the requirement and releases the energy during the period when the supply of energy is less than the requirement. When the flywheel absorbs energy, the speed of flywheel increases where as when the flywheel releases the energy, the speed of flywheel decreases.

Co-efficient of fluctuation of energy :-

→ It is represented by K_e.
→ The ratio of maximum fluctuation of energy to the workdone per cycle is known as co-efficient of fluctuation of energy.

$$K_e = \frac{\text{Max}^m \text{ fluctuation of energy}}{\text{workdone per cycle}}$$

Maximum fluctuation of energy :-

→ The difference between the maximum and minimum energies is known as maximum fluctuation of energy.
→ It is represented by ΔE.

Max^m fluctuation of energy (ΔE) = Max^m energy − Min^m energy

Maximum fluctuation of speed:-

It is the difference between the maximum and minimum speed during a cycle.

Co-efficient of fluctuation of speed:-

The ratio of maximum fluctuation of speed to the mean speed is known as co-efficient of fluctuation of speed.

→ It is denoted by K_s.

$$K_s = \frac{\text{Max}^m \text{ fluctuation of speed}}{\text{Mean speed}}$$

$$= \frac{\text{Max}^m \text{ speed} - \text{Min}^m \text{ speed}}{\text{Mean speed}}$$

N_1 = Max^m speed in rpm during a cycle

N_2 = Min^n speed in rpm during a cycle

N = Mean speed = $\frac{N_1 + N_2}{2}$

$$K_s = \frac{N_1 - N_2}{N} = \frac{N_1 - N_2}{\frac{N_1 + N_2}{2}} = \frac{2(N_1 - N_2)}{N_1 + N_2} = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2}$$

① The maximum and minimum speed of a flywheel are 242 rpm and 238 rpm represented of the flywheel is 2600 kg and radius of gyration is 1.8 m. Find

(i) Mean speed of flywheels

(ii) Max^m fluctuation on energy

(iii) co-efficient of fluctuation of speed.

Ans: Given data,

$N_1 = 242$ rpm

$N_2 = 238$ rpm

$m = 2800$ kg

$r = 1.8$ m

(i) Mean speed of flywheels

$$N = \frac{N_1 + N_2}{2} = \frac{242 + 238}{2} = 240 \text{ rpm (Ans)}$$

(ii) Maximum fluctuation of energy

~~ko~~

(iii) Co-efficient of fluctuation of speed

$$K_s = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2}$$

$$\omega_1 = \frac{2\pi N_1}{60} = \frac{2\pi \times 242}{60} = 25.34 \text{ rad/s}$$

$$\omega_2 = \frac{2\pi N_2}{60} = \frac{2\pi \times 238}{60} = 24.92 \text{ m/s}$$

$$= \frac{2(25.34 - 24.92)}{25.34 + 24.92}$$

$$= 0.016 \text{ (Ans)}$$

Governor	Flywheel
1. Its function is to regulate the supply of driving fluid producing energy. According to the load requirements so that at different loads almost a constant speed is maintained.	1. Its function is to store available mechanical energy when it is in excess of load requirements and to part with the same when the available energy is less than that required by the load.
2. It works intermittently i.e only where there is a change in load.	2. It works continuously from cycle to cycle.
3. It is provided on prime movers such as engines and turbines.	3. It is provided on engine and fabricating M/c viz rolling mills, punching M/c, shear machine.
4. It takes care of fluctuation of speed due to variation of load over long range of working of engines and turbines.	4. In engines it takes care of fluctuation of speed during thermodynamic cycle.
5. Regulates input energy according to the load from cycle to cycle.	5. Regulate the supply of energy during a cycle.
6. Changes the quantity of the working fluid.	6. No control over the quality of working fluid.
7. It is used in automobile vehicles etc.	7. It is used in toys, IC engines etc.

VIBRATIONS

When an elastic body (like shaft, spring etc) which is fixed at one end and is displaced at the other end from its equilibrium position by the application of an external force, the body is said to be in vibrations.

Terms used in vibration :-

1. Time period or period of vibration
2. cycle
3. Frequency
4. Resonance
5. Amplitude

1. Time period :-

It is the time interval after which the motion is repeated it self. The period of vibration is usually expressed in seconds.

2. Cycle :-

It is the motion completed during one period time.

3. Frequency :-

It is the number of cycles completed in one second. It is expressed in Hertz (Hz).

1 Hz = 1 cycle/sec

4. Reasonance :-

When the frequency of external force is the same as that of the natural frequency of the system, reasonance takes place. Reasonance results in large amplitudes of vibrations and this may be dangerous.

5. Amplitude :-

The maximum displacement of a vibrating body from its equilibrium position is known as amplitude.

Types of vibratory motion :-

There are mainly three types of vibratory motion

a. Free/natural vibration
b. Forced vibration
c. Damped vibration

a. Free / natural vibration :- 

When no external force acts on the body after giving it an initial displacement, then the body is said to be under free/natural vibration.

b. Forced vibration :- 

When the body vibrates under influenced by external force, then the body is said to be under forced vibration.

c. Damped vibration :- 

When there is a reduction in amplitude over every cycle of vibration, then the motion is said to be damped vibration. This is due to the fact that a certain amount of energy possessed by the vibrating system is always ~~diss~~ in overcoming frictional resistance to the motion.

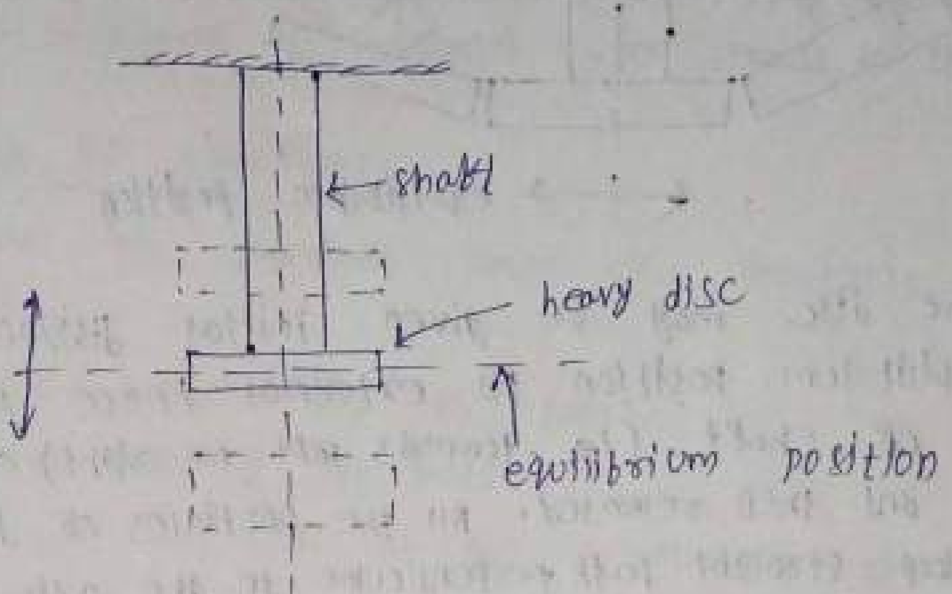
(*) Free vibration :-

In case of free vibration no external force (external torque) acts on the body, after giving an initial displacement. The motion is ~~mentioned~~ maintained by internal elastic forces.

There are mainly three types of free vibrations :-

(a) Longitudinal vibrations
(b) Transverse vibrations
(c) Torsional vibrations.

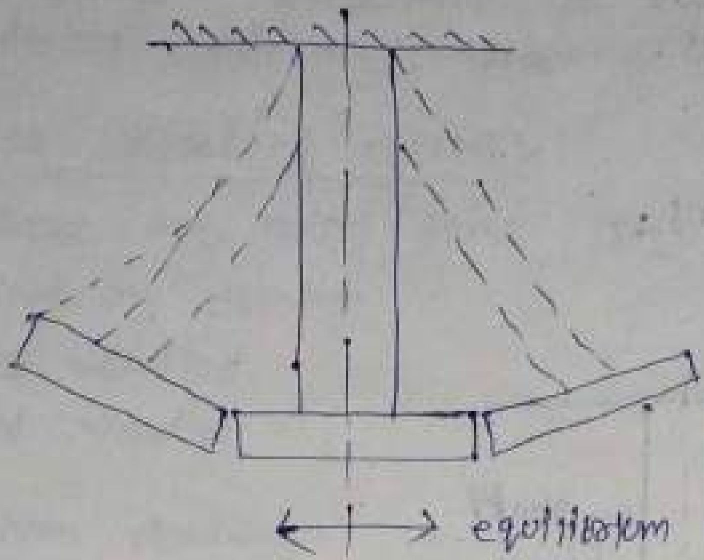
(a) Longitudinal vibrations :-



Consider a weightless vibrating body i.e a shaft or a rod or a spring as shown in figure. whose one end is fixed and other end carries a heavy disc. The shaft is assumed to be weighless.

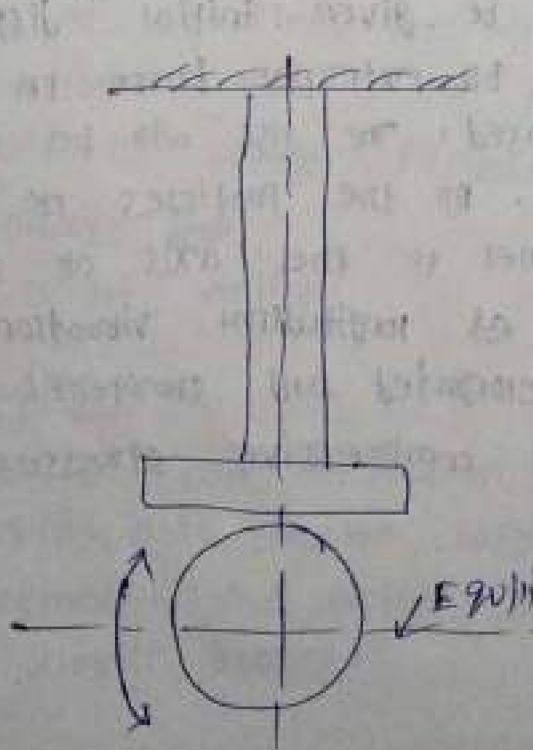
The disc may be given initial displacement from the equilibrium position by external force in the downward direction and then released. The disc will be moving up and down as shown in figure. All the particles of disc will vibrate along straight path parallel to the axis of shaft. This type of vibration is known as longitudinal vibration. In longitudinal vibration, the shaft is elongated and shortened alternately resulting in tensile and compressive stresses in the shaft.

(b) Transverse vibrations :-



The disc may be given initial displacement from equilibrium position by external force perpendicular to axis of shaft (i.e. towards left or right) as shown in figure and then released. All the particles of disc will vibrate along straight path perpendicular to the axis of shaft. This type of vibration is known as transverse vibration. In transverse vibration, the shaft is alternately bent and straight resulting in bending stresses in the shaft.

(c) Torsional vibration :-



The disc may be given initial angular displacement from the equilibrium position by an external torque (i.e the disc may be twisted in a circular arc as shown in fig) and then released. All the particles of disc will vibrate along circular arcs whose centers lie on the axis of shaft. This type of vibration is known as torsional vibration. In torsional vibration, the shaft is alternately twisted and untwisted resulting in torsional shear stress in the shaft.

⊛ Causes of vibrations

Some of important cause of vibrations in machine are:-

a. unbalanced reciprocating M/c parts.
b. unbalanced rotating M/c parts.
c. Incorrect alignment of the transmission elements such as coupling etc.
d. Loose fastenings of moving parts.
e. Loose transmission by belts and chains.
f. worn-out teeth of gears for power transmission.
g. Vibration waves from other sources and M/c installed nearby due to improper isolation of vibrations from them.

⊛ Remedies :-

Although it is impossible to eliminate the vibrations, yet these can be reduced by adopting various remedies. Some of the remedies are:-

a. partial balancing of reciprocating masses.
b. Balancing of unbalanced rotating masses.
c. proper tightening and locking of fastenings periodically and ensuring it again.

d. correcting the misalignment of rotating components and checking it from time to time.

e. Timely replacement of work-out moving parts, slides and bearings with excessive clearance.

f. Isolating vibrations from other M/c and sources by providing vibration insulation pads in the M/c foundations

22.5.23

① ~~A shaft carries four masses A, B, C and D of magnitude 200 kg, 300 kg, 400 kg, 200 kg respectively.~~

① A shaft is rotating at a uniform angular speed. Four masses M_1, M_2, M_3 and M_4 of magnitude 300 kg, 450 kg, 360 kg and 390 kg respectively are attached rigidly to the shaft. The masses are rotating are same plane. The corresponding radius of rotation are 200mm, 150mm, 250mm, 300mm respectively. The angle made by these masses with horizontal 0°, ~~...~~ 45°, 120°, 255° respectively. Find

(i) magnitude of balancing mass

(ii) position of balancing mass if radius of rotation is 200mm.

Ans. Given data,

$M_1 = 300$ kg

$M_2 = 450$ kg

$M_3 = 360$ kg

$M_4 = 390$ kg

$r_1 = 200$ mm $= 0.2$ m

$r_2 = 150$ mm $= 0.15$ m

$r_3 = 250$ mm $= 0.25$ m

$r_4 = 300$ mm $= 0.3$ m

$\theta_1 = 0°$

$\theta_2 = 45°$

$\theta_3 = 120°$

$\theta_4 = 255°$

$$\Sigma H = m_1 r_1 \cos\theta_1 + m_2 r_2 \cos\theta_2 + m_3 r_3 \cos\theta_3 + m_4 r_4 \cos\theta_4$$

$$= (300 \times 0.2 \times \cos 0^\circ) + (450 \times 0.15 \times \cos 45) + (360 \times 0.25 \times \cos 120) + (390 \times 0.3 \times \cos 255)$$

$$= 32.44 \text{ kg.m}$$

$$\Sigma V = m_1 r_1 \sin\theta_1 + m_2 r_2 \sin\theta_2 + m_3 r_3 \sin\theta_3 + m_4 r_4 \sin\theta_4$$

$$= (300 \times 0.2 \times \sin 0^\circ) + (450 \times 0.15 \times \sin 45^\circ) + (360 \times 0.25 \times \sin 120^\circ) + (390 + 0.3 \times \sin 255)$$

$$= 12.65 \text{ kg.m}$$

$$R = \sqrt{\Sigma H^2 + \Sigma V^2}$$

$$= \sqrt{(32.44)^2 + (12.65)^2}$$

$$= 34.81 \text{ kg.m}$$

$$\tan\theta = \frac{\Sigma V}{\Sigma H}$$

$$\tan\theta = \frac{12.65}{32.44} = 0.38$$

$$\Rightarrow \theta' = \tan^{-1}(0.38) = 21^\circ.$$

(ii) $r_2 = 200\text{mm} = 0.2\text{ m}$

we know $m_1 r_1 = m_2 r_2$

$$\Rightarrow 34.81 = m_2 \times 0.2$$

$$\Rightarrow m_2 = \frac{34.81}{0.2} = 174.05$$

Balancing :

Balancing is defined as the process of designing or modifying a machine in which unbalance force is minimum. The rotating reciprocating parts of a high speed engine. If are not properly balanced, the dynamic forces will set up.

These dynamic forces will :-

(i) increase the loads on bearing
(ii) increase the stresses in the members of a machine
(iii) produce unpleasant and even dangerous vibrations.

27.5.23

(1) Four masses A,B,C,D are attached to a shaft and revolve in same plane the masses are 12 kg, 10 kg, 18 kg, 15 kg. respectively, and there radii of rotation are 40 mm, 50 mm, 60 mm and 30 mm. The angular position of masses B, C, D, R 60, 135, 270° from the mass A. Find the magnitude and position of the balancing mass at a radius of 100 mm.

Ans : $M_1 = 12$ kg $\quad r_1 = 40\,mm = 0.04\,m \quad \theta_1 = 0$

$M_2 = 10$ kg $\quad r_2 = 50\,mm = 0.05\,m \quad \theta_2 = 60°$

$M_3 = 18$ kg $\quad r_3 = 60\,mm = 0.06\,m \quad \theta_3 = 135°$

$M_4 = 15$ kg $\quad r_4 = 30\,mm = 0.03\,m \quad \theta_4 = 270°$

$$\Sigma H = M_1 r_1 \cos\theta_1 + M_2 r_2 \cos\theta_2 + M_3 r_3 \cos\theta_3 + M_4 r_4 \cos\theta_4$$

$$= (12 \times 0.04 \times \cos 0) + (10 \times 0.05 \times \cos 60) + (18 \times 0.06 \times \cos 135) + (15 \times 0.03 \times \cos 270)$$

$$= -0.033 \text{ kg.m}$$

$\Sigma V = M_1 r_1 \sin\theta_1 + M_2 r_2 \sin\theta_2 + M_3 r_3 \sin\theta_3 + M_4 r_4 \sin\theta_4$

$= 12 \times 0.04 \times \sin 0 + 10 \times 0.05 \times \sin 60 + 18 \times 0.06 \times \sin 13$

$+ 15 \times 0.03 \times \sin 270$

$= 0.746 \text{ kg.m}$

$R = \sqrt{\Sigma H^2 + \Sigma V^2}$

$= \sqrt{(-0.033)^2 + (0.746)^2}$

$= 0.746 \text{ kg.m}$

$\tan\theta = \dfrac{\Sigma V}{\Sigma H}$

$= \dfrac{0.746}{-0.033} = -22.06$

$\Rightarrow \theta = \tan^{-1}(-22.06) = -87.46$

$M_1 r_1 = M_2 r_2$

$\Rightarrow 0.746 = M_2 \times 0.1$

$\Rightarrow M_2 = \dfrac{0.746}{0.1} = 7.46 \text{ kg.}$